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ORIGINAL ARTICLE

Fractional-order modelling and analysis of diabetes mellitus: Utilizing the Atangana-Baleanu Caputo (ABC) operator

Pooja Yadav [a], Shah Jahan [a,*], Kamal Shah [b,c], Olumuyiwa James Peter [d], Thabet Abdeljawad [b,e,f,g,**]

[a] Department of Mathematics, Central University of Haryana, Mohindergarh, 123031, India
[b] Department of Mathematics and Sciences, Prince Sultan University, Riyadh 11586, Saudi Arabia
[c] Department of Computer Science and Mathematics, Lebanese American University, Byblos, Lebanon
[d] Department of Mathematical and Computer Sciences, University of Medical Sciences, Ondo City, Ondo State, Nigeria
[e] Department of Medical Research, China Medical University, Taichung 40402, Taiwan
[f] Department of Mathematics Kyung Hee University, 26 Kyungheedae-ro, Dongdaemun-gu, Seoul 02447, Republic of Korea
[g] Department of Mathematics and Applied Mathematics, School of Science and Technology, Sefako Makgatho Health Sciences University, Ga-Rankuwa, Medusa 0204, South Africa

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A B S T R A C T

This article aims to introduce and analyze a diabetes mellitus model of fractional order, utilizing the ABC derivative. Diabetes mellitus is a prevalent and significant disease worldwide, ranking among the top causes of mortality. It is characterized by chronic metabolic dysfunction, leading to elevated blood glucose levels and subsequent damage to vital organs including the nerves, kidneys, eyes, blood vessels, and heart. The fractional ABC derivative is used in this study to describe and analyze diabetes mellitus mathematically while removing hereditary influences. The investigation begins by exploring the initial points of the diabetes mellitus model. Under the fractional ABC operator, Picard's theorem is used to prove the existence and uniqueness of solutions. For the numerical approximation of solutions in the fractional-order diabetes mellitus model, this study used a specialized technique that combines the principles of fractional calculus and a two-step Lagrange polynomial interpolation. Finally, the obtained results are visually presented through graphical representations, serving as empirical evidence to support our theoretical findings. The numerical experiments showed that the proportion of patients with diabetes mellitus increased as the fractional dimension (θ) reduced. The combination of mathematical modelling, analysis, and numerical simulations provides insights into the dynamics of diabetes mellitus, offering valuable contributions to the understanding and management of this prevalent disease. Additionally, the proposed scheme can be enhanced by incorporating the ABC operator, allowing for the simulation of real-world dynamics and behavior in the coexistence of diabetes mellitus and tuberculosis.

1. Introduction

Diabetes mellitus, a chronic metabolic disorder, continues to be a significant global health concern. It is a chronic metabolic condition characterized by persistent hyperglycemia caused by deficiencies in insulin synthesis, action, or both. These abnormalities disrupt the metabolism of carbohydrates, lipids, and proteins, which highlights the crucial role of insulin as an anabolic hormone. Diabetes patients are four times more likely to have a stroke than people without the disease, and they have a greater chance of developing coronary artery disease. Among the risky effects of untreated diabetes are visual abnormalities that can lead to blindness, loss of consciousness, and susceptibility to infections. Conversely, others, particularly children with a complete deficiency of insulin, may experience noticeable symptoms such as excessive urination (polyuria), excessive thirst (polydipsia), increased appetite (polyphagia), unintended weight loss, and blurred vision. In 2015, about 415 million people aged 20 to 79 had diabetes mellitus, and the International Diabetes Federation (IDF) predicts that figure

* Corresponding author at: Department of Mathematics, Central University of Haryana, Mohindergarh 123031, India.
** Corresponding author at: Department of Mathematics and Sciences, Prince Sultan University, Riyadh 11586, Saudi Arabia.
E-mail addresses: shahjahan@cuh.ac.in (S. Jahan), tabdeljawad@psu.edu.sa (T. Abdeljawad).

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would increase by another 200 million by 2040 [2]. The etiology of diabetes mellitus encompasses a diverse array of genetic susceptibilities, including variations in genes involved in insulin production, secretion, and regulation. The pathophysiology of diabetes mellitus involves complex disturbances in glucose metabolism, lipid metabolism, and protein homeostasis. As our understanding of diabetes mellitus continues to evolve, extensive research efforts have been dedicated to unraveling its etiology, exploring the underlying pathophysiological mechanisms, and developing effective therapeutic strategies [6].

In recent years, infectious disease modelling has gained significant attention as it strives to provide solutions to epidemics and plagues that have plagued humanity. Numerous studies have explored integer-order infection models in the literature. In research on an integer-order COVID-19 and Dengue co-infection model, Omame et al. [5] emphasized the significance of preventative actions for either illness in Brazil. The authors in [3–5] focused on an integer-order co-infection model of Zika virus and dengue fever, Cholera and Buruli ulcer, syphilis, and HPV with optimal control. However, there are limitations in the above mentioned models as they fail to incorporate memory, a crucial aspect in accurately representing real-life scenarios [1].

In the field of scientific research, it has been widely recognized over the past few decades that fractional models provide a more accurate representation of natural phenomena compared to traditional differential equations of integer order. This advantage has led to the increasing importance and popularity of fractional calculus, particularly in modelling real-world scenarios with memory effects. Numerous studies have been conducted on theoretical and numerical approaches for fractional-order systems as a result of the helpfulness of fractional calculus in a variety of fields, including engineering, biology, and the social sciences [7,14–16,32–34]. Different kinds of fractional operators are extremely important in the field of fractional calculus for improving our comprehension of model behavior. These operators include Atangana-Baleanu, Caputo-Fabrizio, Caputo, Riemann-Liouville, and many others, each offering distinct advantages and disadvantages [17,18,23–26,28–31]. Riemann-Liouville operators, for example, are employed to solve certain models; however, they pose challenges due to the requirement of fractional order conditions. In contrast, the Caputo fractional operator overcomes this limitation by allowing the use of initial conditions with integer-order derivatives. This characteristic grants these initial conditions a clear and discernible interpretation [19,20].

In this study, we begin by establishing a mathematical model using integer order derivatives. Subsequently, we employ the Atangana-Baleanu fractional operator to further enhance our analysis. The rationale behind selecting the Atangana-Baleanu operator lies in its unique characteristics, including a nonlocal and nonsingular kernel represented by the Mittag-Leffler function. This operator is well-suited for capturing the complex dynamics inherent in the model under investigation. Notably, previous research has explored the Atangana-Baleanu operator and its applications in diverse systems within the realms of applied sciences and engineering, as documented in references [8–12]. These studies provide valuable insights into the efficacy and versatility of the Atangana-Baleanu operator in various contexts. Inspired by the successful utilization of fractional operators in various real-life scenarios, this study aims to apply the concept of the fractional operator of order (θ) to model the differential equations of diabetes mellitus using the ABC operator. Our objective is to investigate the impact of the fractional order on the dynamics of each subclass within the population, providing valuable insights into the behavior of diabetes mellitus. However, there are no direct limitations, but the ABC fractional derivatives work in a restricted space. There are fractional differential equations that the ABC derivative cannot solve.

The structure of this article is: Section 2 focuses on the mathematical modelling of diabetes mellitus, presenting the formulation and key considerations in our model. Moving forward, section 3 provides an in-depth analysis of the existence and uniqueness of solutions. In section 4, we conduct numerical simulations and engage in a comprehensive discussion of the results obtained. Lastly, in the concluding section of this paper, we provide a comprehensive summary of the primary findings and discuss the implications that arise from our study.

1.1. Preliminaries

This subsection introduces the fundamental theory of the ABC derivative, laying the groundwork for its application in the subsequent sections.

Definition 1.1. On the interval $\theta \in [0,1]$, by taking the function $f \in H^1(a,b), b > a$, so the ABC operator is given by

$$ {}_{a}^{ABC}D_{\lambda}^{\theta} f(\lambda) = \frac{M(\theta)}{1-\theta} \int_{a}^{\lambda} f'(y) E_{\theta} \left[-\theta \frac{(\lambda-\theta)^{\theta}}{1-\theta} \right] \mathrm{d}y, $$

with $M(0) = M(1) = 1$ [13], here $M(\theta)$ is a normalization function.

Definition 1.2. [23] On the interval $\theta \in [0,1]$, by taking the function $f \in H^1(a,b), b > a$, which is not differentiable, therefore the Atangana-Baleanu fractional operator in Riemann-Liouville sense is defined as

$$ {}_{a}^{ABR}D_{\lambda}^{\theta} f(\lambda) = \frac{M(\theta)}{1-\theta} \frac{\mathrm{d}}{\mathrm{d}\lambda} \int_{a}^{\lambda} f(y) E_{\theta} \left[-\theta \frac{(\lambda-\theta)^{\theta}}{1-\theta} \right] \mathrm{d}y, $$

Definition 1.3. [23] The ABC fractional operator for the fractional integral of order θ is given by

$$ {}_{a}^{AB}I_{\lambda}^{\theta} f(\lambda) = \frac{1-\theta}{\mathcal{A}(\theta)} f(t) + \frac{\theta}{\mathcal{A}(\theta)\Gamma(\theta)} \int_{a}^{\lambda} f(y)(\lambda-y)^{\theta-1}\, \mathrm{d}y. $$

If $\theta = 0$ and $\theta = 1$ the initial function and ordinary integral are obtained respectively.

In the subsequent sections, we will examine the Laplace transform operators and explore the fundamental theorems associated with these derivatives. We will also explore the connection between these operators and the Laplace transform, specifically focusing on its representation in the following form.

$$ \mathcal{L}\left\{ {}_{0}^{ABR}D_{\lambda}^{\theta}[f(\lambda)] \right\}(l) = \frac{L(\theta)}{1-\theta} \frac{l^{\theta}\mathcal{L}\{f(\lambda)\}(l)}{l^{\theta} + \frac{\theta}{1-\theta}} $$

and

$$ \mathcal{L}\left\{ {}_{0}^{ABC}D_{\lambda}^{\theta}[f(\lambda)] \right\}(l) = \frac{M(\theta)}{1-\theta} \frac{l^{\theta}\mathcal{L}\{f(\lambda)\}(l) - l^{\theta-1}f(0)}{l^{\theta} + \frac{\theta}{1-\theta}}. $$

Theorem 1.4. *[23] Take into account a closed interval [a, b] and use g to represent a continuous function defined on it. We can prove the following inequality, which is true for any point inside of [a, b]:*

$$ \left\| {}_{0}^{ABR}D_{\lambda}^{\theta}[f(\lambda)] \right\| < \frac{M(\theta)}{1-\theta} \|f(\xi)\|, $$

where $\|f(\xi)\| = \max_{a \leqslant \lambda \leqslant b} |f(\xi)|$.

Theorem 1.5. *[13] The Riemann-Liouville and Caputo types of the Atangana-Baleanu derivative exhibit the Lipschitz condition, which is best defined as follows:*

$$ \left\| {}_{0}^{ABR}D_{\lambda}^{\theta}[f(\lambda)] - {}_{0}^{ABR}D_{\lambda}^{\theta}[g(\lambda)] \right\| \leqslant H\|f(\lambda) - g(\lambda)\|, $$

and

$$ \left\| {}_{0}^{ABC}D_{\lambda}^{\theta}[f(\lambda)] - {}_{0}^{ABC}D_{\lambda}^{\theta}[g(\lambda)] \right\| \leqslant H\|f(\lambda) - g(\lambda)\|. $$

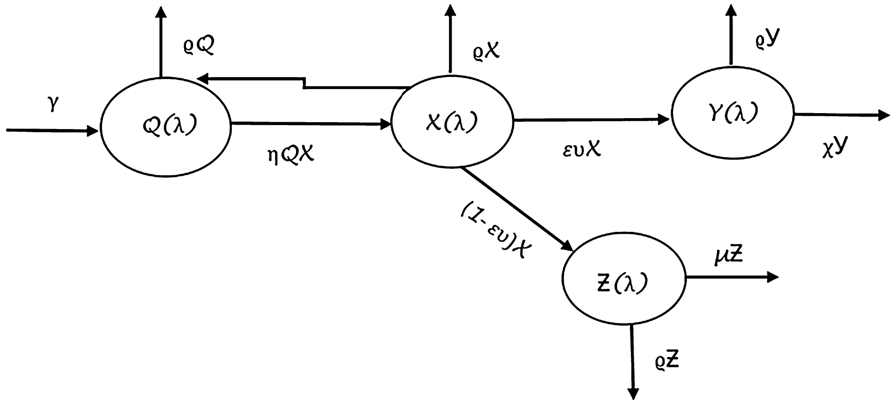

Fig. 1. Flowchart of the diabetes mellitus model.

Theorem 1.6. *[27] Let $(X, ||\cdot||)$ be a Banach space and $T : X \to X$ be a contraction on X, meaning there exists a constant $a \in (0,1)$ such that $|T(x) - T(y)| \leqslant a|x - y|$ for all $x, y \in X$. Then:*
i. T has a fixed point $x^* \in X$, i.e., $T(x^*) = x^*$.
ii. For a sequence $\{x_n\}_{n=0}^{\infty}$ defined by $x_{n+1} = T(x_n)$, for $n = 0, 1, 2, 3, \ldots$, it converges to x^*.

2. Mathematical model

This section aims to provide a mathematical formulation for the diabetes mellitus model, which has attracted significant attention in the scientific literature [21,22]. The development of accurate mathematical models holds the utmost importance in predicting the dynamics and components of diabetes. The motivation for this study arises from the profound and far-reaching impact of diabetes on human life, particularly through its associated complications. Hence, it becomes imperative to undertake a comprehensive examination and analysis of diabetes models in order to enhance our comprehension of the underlying mechanisms of the disease and enable accurate predictions concerning its behavior. Consequently, this study specifically focuses on investigating the diabetes mellitus model that considers treatment and excludes genetic factors. Through the exploration and evaluation of this model, our aim is to gain valuable insights into the dynamics and management of diabetes mellitus.

$$\begin{aligned}
\frac{d\mathcal{Q}(\lambda)}{d\lambda} &= \gamma - \varrho\mathcal{Q} - \eta\mathcal{Q}\mathcal{X},\\
\frac{d\mathcal{X}(\lambda)}{d\lambda} &= \eta\mathcal{Q}\mathcal{X} - (\varrho+1)\mathcal{X},\\
\frac{d\mathcal{Y}(\lambda)}{d\lambda} &= \varepsilon\upsilon\mathcal{X} - (\varrho+\chi)\mathcal{Y},\\
\frac{d\mathcal{Z}(\lambda)}{d\lambda} &= (1-\varepsilon\upsilon)\mathcal{X} - (\varrho+\mu)\mathcal{Z},\\
\mathcal{W}(\lambda) &= \mathcal{Q}(\lambda) + \mathcal{X}(\lambda) + \mathcal{Y}(\lambda) + \mathcal{Z}(\lambda),
\end{aligned} \tag{2.1}$$

with initial conditions
$\mathcal{Q}(0) = \mathcal{Q}_0$, $\mathcal{X}(0) = \mathcal{X}_0$, $\mathcal{Y}(0) = \mathcal{Y}_0$, $\mathcal{Z}(0) = \mathcal{Z}_0$, $\mathcal{W}(0) = \mathcal{W}_0$,
where, the class $\mathcal{Q}(\lambda)$ denotes the group of susceptible individuals who are at risk of contracting the disease, $\mathcal{W}(\lambda)$ is susceptible population, $\mathcal{X}(\lambda)$ is exposed population, $\mathcal{Y}(\lambda)$ is infected population, $\mathcal{Z}(\lambda)$ is recovered population by treatment (Fig. 1).
We can write the last equation of the given system in terms of others as:

$$\frac{d\mathcal{W}(\lambda)}{d\lambda} = \frac{d\mathcal{Q}(\lambda)}{d\lambda} + \frac{d\mathcal{X}(\lambda)}{d\lambda} + \frac{d\mathcal{Y}(\lambda)}{d\lambda} + \frac{d\mathcal{Z}(\lambda)}{d\lambda},$$

this above equation yields,

$$\frac{d\mathcal{W}(\lambda)}{d\lambda} = \gamma - \varrho\mathcal{W} - \chi\mathcal{Y} - \mu\mathcal{Z}.$$

This resulting in simpler form of diabetes mellitus model as:

$$\begin{aligned}
\frac{d\mathcal{W}(\lambda)}{d\lambda} &= \gamma - \varrho\mathcal{W} - \chi\mathcal{Y} - \mu\mathcal{Z},\\
\frac{d\mathcal{X}(\lambda)}{d\lambda} &= \eta(\mathcal{W} - \mathcal{X} - \mathcal{Y} - \mathcal{Z})\mathcal{X} - (\varrho+1)\mathcal{X},\\
\frac{d\mathcal{Y}(\lambda)}{d\lambda} &= \varepsilon\upsilon\mathcal{X} - (\varrho+\chi)\mathcal{Y},\\
\frac{d\mathcal{Z}(\lambda)}{d\lambda} &= (1-\varepsilon\upsilon)\mathcal{X} - (\varrho+\mu)\mathcal{Z},
\end{aligned} \tag{2.2}$$

where (Table 1)

Table 1
Description of parameters.

Parameter	Description
γ	births in a given time range,
ϱ	deaths in a given time range,
χ	disease-specific fatality rate,
μ	disease-related mortality rate with treatment,
$\varepsilon\upsilon$	rate of transition from latent to infected individuals without treatment,
η	rate of transmission from exposed to susceptible individuals.

Let us define the fractional order model of (2.2)

$$\begin{aligned}
{}_0^{ABC}D_\lambda^\theta\mathcal{W}(\lambda) &= \gamma - \varrho\mathcal{W} - \chi\mathcal{Y} - \mu\mathcal{Z},\\
{}_0^{ABC}D_\lambda^\theta\mathcal{X}(\lambda) &= \eta(\mathcal{W} - \mathcal{X} - \mathcal{Y} - \mathcal{Z})\mathcal{X} - (\varrho+1)\mathcal{X},\\
{}_0^{ABC}D_\lambda^\theta\mathcal{Y}(\lambda) &= \varepsilon\upsilon\mathcal{X} - (\varrho+\chi)\mathcal{Y},\\
{}_0^{ABC}D_\lambda^\theta\mathcal{Z}(\lambda) &= (1-\varepsilon\upsilon)\mathcal{X} - (\varrho+\mu)\mathcal{Z}.
\end{aligned} \tag{2.3}$$

3. Existence and uniqueness analysis

Solving nonlinear equations is known to be a challenging topic in differential calculus. The fractional model addressed in this study is both nonlocal and nonlinear, making it infeasible to obtain exact solutions for these systems. Hence, our focus lies on investigating the existence and uniqueness of solutions for the fractional model (2.3). In order to accomplish this, we employ the fixed-point theorem.

On the interval q, suppose that $P = K(q) \times K(q)$, where the Banach space $K(q)$ of continuous real values functions is defined with the norm

$$\|\mathcal{W}, \mathcal{X}, \mathcal{Y}, \mathcal{Z}\| = \|\mathcal{W}\| + \|\mathcal{X}\| + \|\mathcal{Y}\| + \|\mathcal{Z}\|,$$

where,

$$\begin{aligned}
\|\mathcal{W}\| &= \sup\{|\mathcal{W}(\lambda)| : \lambda \in q\},\\
\|\mathcal{X}\| &= \sup\{|\mathcal{X}(\lambda)| : \lambda \in q\},\\
\|\mathcal{Y}\| &= \sup\{|\mathcal{Y}(\lambda)| : \lambda \in q\},\\
\|\mathcal{Z}\| &= \sup\{|\mathcal{Z}(\lambda)| : \lambda \in q\}.
\end{aligned}$$

The system (2.2) is transformed into the following equation by taking Atangana-Baleanu fractional integral into account:

$$\begin{aligned}
\mathcal{W}(\lambda)-\mathcal{W}(0)&=\frac{1-\theta}{\mathcal{A}(\theta)}\{\gamma-\varrho\mathcal{W}-\chi\mathcal{Y}-\mu\mathcal{Z}\}\\
&+\frac{\theta}{\mathcal{A}(\theta)\Gamma(\theta)}\int_0^\lambda(\lambda-y)^{\theta-1}\{\gamma-\varrho\mathcal{W}-\chi\mathcal{Y}-\mu\mathcal{Z}\}dy,\\
\mathcal{X}(\lambda)-\mathcal{X}(0)&=\frac{1-\theta}{\mathcal{A}(\theta)}\{\eta(\mathcal{W}-\mathcal{X}-\mathcal{Y}-\mathcal{Z})\mathcal{X}-(\varrho+1)\mathcal{X}\}\\
&+\frac{\theta}{\mathcal{A}(\theta)\Gamma(\theta)}\int_0^\lambda(\lambda-y)^{\theta-1}\{\eta(\mathcal{W}-\mathcal{X}-\mathcal{Y}-\mathcal{Z})\mathcal{X}-(\varrho+1)\mathcal{X}\}dy,\\
\mathcal{Y}(\lambda)-\mathcal{Y}(0)&=\frac{1-\theta}{\mathcal{A}(\theta)}\{\varepsilon\upsilon\mathcal{X}-(\varrho+\chi)\mathcal{Y}\}\\
&+\frac{\theta}{\mathcal{A}(\theta)\Gamma(\theta)}\int_0^\lambda(\lambda-y)^{\theta-1}\{\varepsilon\upsilon\mathcal{X}-(\varrho+\chi)\mathcal{Y}\}dy,\\
\mathcal{Z}(\lambda)-\mathcal{Z}(0)&=\frac{1-\theta}{\mathcal{A}(\theta)}\{(1-\varepsilon\upsilon)\mathcal{X}-(\varrho+\mu)\mathcal{Z}\}\\
&+\frac{\theta}{\mathcal{A}(\theta)\Gamma(\theta)}\int_0^\lambda(\lambda-y)^{\theta-1}\{(1-\varepsilon\upsilon)\mathcal{X}-(\varrho+\mu)\mathcal{Z}\}dy.
\end{aligned}\tag{3.1}$$

We simplify (3.1) by writing

$$\begin{aligned}
K_1(\lambda,\mathcal{W})&=\gamma-\varrho\mathcal{W}-\chi\mathcal{Y}-\mu\mathcal{Z},\\
K_2(\lambda,\mathcal{X})&=\eta(\mathcal{W}-\mathcal{X}-\mathcal{Y}-\mathcal{Z})\mathcal{X}-(\varrho+1)\mathcal{X},\\
K_3(\lambda,\mathcal{Y})&=\varepsilon\upsilon\mathcal{X}-(\varrho+\chi)\mathcal{Y},\\
K_4(\lambda,\mathcal{Z})&=(1-\varepsilon\upsilon)\mathcal{X}-(\varrho+\mu)\mathcal{Z}.
\end{aligned}$$

Theorem 3.1. *If the aforementioned inequality holds:*

$0\leqslant\beta_1<1,$

$0\leqslant\beta_2<1,$

$0\leqslant\beta_3<1,$

$0\leqslant\beta_4<1.$

Then the kernels $\mathcal{K}_1,\mathcal{K}_2,\mathcal{K}_3$ and $\mathcal{K}_4$ satisfy the Lipschitz condition and contraction.

Proof. By taking the kernel $\mathcal{K}_1(\lambda,\mathcal{W})=\gamma-\varrho\mathcal{W}-\chi\mathcal{Y}-\mu\mathcal{Z}$. Let $\mathcal{W}$ and $\mathcal{W}_1$ be two functions, so we have:

$$\begin{aligned}
\left\|\mathcal{K}_1(\lambda,\mathcal{W}(\lambda))-\mathcal{K}_1\left(\lambda,\mathcal{W}_1(\lambda)\right)\right\|&=||\varrho(\mathcal{W}-\mathcal{W}_1)||,\\
&\leqslant\varrho||\mathcal{W}-\mathcal{W}_1||,\\
&\leqslant\beta_1\left\|\mathcal{W}-\mathcal{W}_1\right\|.
\end{aligned}$$

Taking $\beta_1=\varrho$, where $P_1=\max_{\lambda\in J}\|\mathcal{W}(\lambda)\|, P_2=\max_{\lambda\in J}\|\mathcal{X}(\lambda)\|, P_3=\max_{\lambda\in J}\|\mathcal{Y}(\lambda)\|, P_4=\max_{\lambda\in J}\|\mathcal{Z}(\lambda)\|$ are bounded function, then we get

$$\left\|\mathcal{K}_1(\lambda,\mathcal{W}(\lambda))-\mathcal{K}_1\left(\lambda,\mathcal{W}_1(\lambda)\right)\right\|\leqslant\beta_1\left\|\mathcal{W}(\lambda)-\mathcal{W}_1(\lambda)\right\|.$$

Thus, $\mathcal{K}_1$ satisfied the Lipschitz condition, and if $0\leqslant\beta_1<1$, then it is also a contraction for $\mathcal{K}_1$. In same manner, the Lipschitz condition is satisfied by other kernels:

$$\begin{aligned}
\left\|\mathcal{K}_2(\lambda,\mathcal{X}(\lambda))-\mathcal{K}_2\left(\lambda,\mathcal{X}_1(\lambda)\right)\right\|&\leqslant\beta_2\left\|\mathcal{X}(\lambda)-\mathcal{X}_1(\lambda)\right\|,\\
\left\|\mathcal{K}_3(\lambda,\mathcal{Y}(\lambda))-\mathcal{K}_3\left(\lambda,\mathcal{Y}_1(\lambda)\right)\right\|&\leqslant\beta_3\left\|\mathcal{Y}(\lambda)-\mathcal{Y}_1(\lambda)\right\|,\\
\left\|\mathcal{K}_4(\lambda,\mathcal{Z}(\lambda))-\mathcal{K}_4\left(\lambda,\mathcal{Z}_1(\lambda)\right)\right\|&\leqslant\beta_4\left\|\mathcal{Z}(\lambda)-\mathcal{Z}_1(\lambda)\right\|.
\end{aligned}$$

By taking the kernels for the model in consideration, (3.1) can be presented by:

$$\begin{aligned}
\mathcal{W}(\lambda)&=\mathcal{W}(0)+\frac{1-\theta}{\mathcal{A}(\theta)}\mathcal{K}_1(\lambda,\mathcal{W})+\frac{\theta}{\mathcal{A}(\theta)\Gamma(\theta)}\int_0^\lambda(\lambda-y)^{\theta-1}\mathcal{K}_1(y,\mathcal{W})dy,\\
\mathcal{X}(\lambda)&=\mathcal{X}(0)+\frac{1-\theta}{\mathcal{A}(\theta)}\mathcal{K}_2(\lambda,\mathcal{X})+\frac{\theta}{\mathcal{A}(\theta)\Gamma(\theta)}\int_0^\lambda(\lambda-y)^{\theta-1}\mathcal{K}_2(y,\mathcal{X})dy,\\
\mathcal{Y}(\lambda)&=\mathcal{Y}(0)+\frac{1-\theta}{\mathcal{A}(\theta)}\mathcal{K}_3(\lambda,\mathcal{Y})+\frac{\theta}{\mathcal{A}(\theta)\Gamma(\theta)}\int_0^\lambda(\lambda-y)^{\theta-1}\mathcal{K}_3(y,\mathcal{Y})dy,\\
\mathcal{Z}(\lambda)&=\mathcal{Z}(0)+\frac{1-\theta}{\mathcal{A}(\theta)}\mathcal{K}_4(\lambda,\mathcal{Z})+\frac{\theta}{\mathcal{A}(\theta)\Gamma(\theta)}\int_0^\lambda(\lambda-y)^{\theta-1}\mathcal{K}_4(y,\mathcal{Z})dy.
\end{aligned}\tag{3.2}$$

Therefore, we can present recursively as:

$$\begin{aligned}
\mathcal{W}_\upsilon(\lambda)&=\frac{1-\theta}{\mathcal{A}(\theta)}\mathcal{K}_1\left(\lambda,\mathcal{W}_{\upsilon-1}\right)+\frac{\theta}{\mathcal{A}(\theta)\Gamma(\theta)}\int_0^\lambda(\lambda-y)^{\theta-1}\mathcal{K}_1\left(y,\mathcal{W}_{\upsilon-1}\right)dy,\\
\mathcal{X}_\upsilon(\lambda)&=\frac{1-\theta}{\mathcal{A}(\theta)}\mathcal{K}_2\left(\lambda,\mathcal{X}_{\upsilon-1}\right)+\frac{\theta}{\mathcal{A}(\theta)\Gamma(\theta)}\int_0^\lambda(\lambda-y)^{\theta-1}\mathcal{K}_2\left(y,\mathcal{X}_{\upsilon-1}\right)dy,\\
\mathcal{Y}_\upsilon(\lambda)&=\frac{1-\theta}{\mathcal{A}(\theta)}\mathcal{K}_3\left(\lambda,\mathcal{Y}_{\upsilon-1}\right)+\frac{\theta}{\mathcal{A}(\theta)\Gamma(\theta)}\int_0^\lambda(\lambda-y)^{\theta-1}\mathcal{K}_3\left(y,\mathcal{Y}_{\upsilon-1}\right)dy,\\
\mathcal{Z}_\upsilon(\lambda)&=\frac{1-\theta}{\mathcal{A}(\theta)}\mathcal{K}_4\left(\lambda,\mathcal{Z}_{\upsilon-1}\right)+\frac{\theta}{\mathcal{A}(\theta)\Gamma(\theta)}\int_0^\lambda(\lambda-y)^{\theta-1}\mathcal{K}_4\left(y,\mathcal{Z}_{\upsilon-1}\right)dy.
\end{aligned}$$

With the defined initial conditions as:

$\mathcal{W}_0(\lambda)=\mathcal{W}(0),$

$\mathcal{X}_0(\lambda)=\mathcal{X}(0),$

$\mathcal{Y}_0(\lambda)=\mathcal{Y}(0),$

$\mathcal{Z}_0(\lambda)=\mathcal{Z}(0).$

The following system of equations is formed by using the initial conditions and the difference between the succeeding terms.

$$\begin{aligned}
\Delta_\upsilon(\lambda)=\mathcal{W}_\upsilon(\lambda)-\mathcal{W}_{\upsilon-1}(\lambda)&=\frac{1-\theta}{\mathcal{A}(\theta)}\left(\mathcal{K}_1\left(\lambda,\mathcal{W}_{\upsilon-1}\right)-\mathcal{K}_1\left(\lambda,\mathcal{W}_{\upsilon-2}\right)\right)\\
&+\frac{\theta}{\mathcal{A}(\theta)\Gamma(\theta)}\int_0^\lambda(\lambda-y)^{\theta-1}\left(\mathcal{K}_1\left(y,\mathcal{W}_{\upsilon-1}\right)-\mathcal{K}_1\left(y,\mathcal{W}_{\upsilon-2}\right)\right)dy,\\
\aleph_\upsilon(\lambda)=\mathcal{X}_\upsilon(\lambda)-\mathcal{X}_{\upsilon-1}(\lambda)&=\frac{1-\theta}{\mathcal{A}(\theta)}\left(\mathcal{K}_2\left(\lambda,\mathcal{X}_{\upsilon-1}\right)-\mathcal{K}_2\left(\lambda,\mathcal{X}_{\upsilon-2}\right)\right)+\\
&\frac{\theta}{\mathcal{A}(\theta)\Gamma(\theta)}\int_0^\lambda(\lambda-y)^{\theta-1}\left(\mathcal{K}_2\left(y,\mathcal{X}_{\upsilon-1}\right)-\mathcal{K}_2\left(y,\mathcal{X}_{\upsilon-2}\right)\right)dy,\\
\Psi_\upsilon(\lambda)=\mathcal{Y}_\upsilon(\lambda)-\mathcal{Y}_{\upsilon-1}(\lambda)&=\frac{1-\theta}{\mathcal{A}(\theta)}\left(\mathcal{K}_3\left(\lambda,\mathcal{Y}_{\upsilon-1}\right)-\mathcal{K}_3\left(\lambda,\mathcal{Y}_{\upsilon-2}\right)\right)+\\
&\frac{\theta}{\mathcal{A}(\theta)\Gamma(\theta)}\int_0^\lambda(\lambda-y)^{\theta-1}\left(\mathcal{K}_3\left(y,\mathcal{Y}_{\upsilon-1}\right)-\mathcal{K}_3\left(y,\mathcal{Y}_{\upsilon-2}\right)\right)dy,\\
F_\upsilon(\lambda)=\mathcal{Z}_\upsilon(\lambda)-\mathcal{Z}_{\upsilon-1}(\lambda)&=\frac{1-\theta}{\mathcal{A}(\theta)}\left(\mathcal{K}_4\left(\lambda,\mathcal{Z}_{\upsilon-1}\right)-\mathcal{K}_4\left(\lambda,\mathcal{Z}_{\upsilon-2}\right)\right)+\\
&\frac{\theta}{\mathcal{A}(\theta)\Gamma(\theta)}\int_0^\lambda(\lambda-y)^{\theta-1}\left(\mathcal{K}_4\left(y,\mathcal{Z}_{\upsilon-1}\right)-\mathcal{K}_4\left(y,\mathcal{Z}_{\upsilon-2}\right)\right)dy.
\end{aligned}\tag{3.3}$$

It should be noted that

$$\mathcal{W}_\upsilon(\lambda)=\sum_{i=1}^{\upsilon}\Delta_i(\lambda),\quad \mathcal{X}_\upsilon(\lambda)=\sum_{i=1}^{\upsilon}\aleph_i(\lambda),\quad \mathcal{Y}_\upsilon(\lambda)=\sum_{i=1}^{\upsilon}\Psi_i(\lambda),\quad \mathcal{Z}_\upsilon(\lambda)=\sum_{i=1}^{\upsilon}F_i(\lambda). \tag{3.4}$$

By taking (3.3), the triangular inequality is taken into account after applying the norm on (3.4) the equation transformed into (3.5),

$$\begin{aligned}\left\|\Delta_\upsilon(\lambda)\right\|&=\left\|\mathcal{W}_\upsilon(\lambda)-\mathcal{W}_{\upsilon-1}(\lambda)\right\|,\\ &\leqslant\frac{1-\theta}{\mathcal{A}(\theta)}\left\|\mathcal{K}_1\left(\lambda,\mathcal{W}_{\upsilon-1}\right)-\mathcal{K}_1\left(\lambda,\mathcal{W}_{\upsilon-2}\right)\right\|\\ &\quad+\frac{\theta}{\mathcal{A}(\theta)\Gamma(\theta)}\left\|\int_0^\lambda(\lambda-y)^{\theta-1}\left(\mathcal{K}_1\left(y,\mathcal{W}_{\upsilon-1}\right)-\mathcal{K}_1\left(y,\mathcal{W}_{\upsilon-2}\right)\right)\mathrm{d}y\right\|.\end{aligned} \tag{3.5}$$

As the Lipschitz condition is satisfied by the kernel, the following equation is obtained;

$$\left\|\mathcal{W}_\upsilon(\lambda)-\mathcal{W}_{\upsilon-1}(\lambda)\right\|\leqslant\frac{1-\theta}{\mathcal{A}(\theta)}\beta_1\left\|\mathcal{W}_{\upsilon-1}-\mathcal{W}_{\upsilon-2}\right\|+\frac{\theta}{\mathcal{A}(\theta)\Gamma(\theta)}\beta_1\int_0^\lambda(\lambda-y)^{\theta-1}\left\|\mathcal{W}_{\upsilon-1}-\mathcal{W}_{\upsilon-2}\right\|dy.$$

Also,

$$\left\|\Delta_\upsilon(\lambda)\right\|\leqslant\frac{1-\theta}{\mathcal{A}(\theta)}\beta_1\left\|\Delta_{\upsilon-1}(\lambda)\right\|+\frac{\theta}{\mathcal{A}(\theta)\Gamma(\theta)}\beta_1\int_0^\lambda(\lambda-y)^{\theta-1}\left\|\Delta_{\upsilon-1}(y)\right\|\mathrm{d}y. \tag{3.6}$$

In same manner we derived the following results:

$$\left\|\aleph_\upsilon(\lambda)\right\|\leqslant\frac{1-\theta}{\mathcal{A}(\theta)}\beta_2\left\|\aleph_{\upsilon-1}(\lambda)\right\|+\frac{\theta}{\mathcal{A}(\theta)\Gamma(\theta)}\beta_2\int_0^\lambda(\lambda-y)^{\theta-1}\left\|\aleph_{\upsilon-1}(y)\right\|\mathrm{d}y,$$

$$\left\|\Psi_\upsilon(\lambda)\right\|\leqslant\frac{1-\theta}{\mathcal{A}(\theta)}\beta_3\left\|\Psi_{\upsilon-1}(\lambda)\right\|+\frac{\theta}{\mathcal{A}(\theta)\Gamma(\theta)}\beta_3\int_0^\lambda(\lambda-y)^{\theta-1}\left\|\Psi_{\upsilon-1}(y)\right\|\mathrm{d}y,$$

$$\left\|F_\upsilon(\lambda)\right\|\leqslant\frac{1-\theta}{\mathcal{A}(\theta)}\beta_4\left\|F_{\upsilon-1}(\lambda)\right\|+\frac{\theta}{\mathcal{A}(\theta)\Gamma(\theta)}\beta_4\int_0^\lambda(\lambda-y)^{\theta-1}\left\|F_{\upsilon-1}(y)\right\|\mathrm{d}y. \quad\square$$

Theorem 3.2. *A unique solution is possessed by the proposed fractional order diabetes mellitus model with ABC operator (2.2) having the following condition that is satisfying the* $\lambda_{\max}$ *as*

$$\frac{1-\theta}{\mathcal{A}(\theta)}\beta_i+\frac{\lambda_{\max}^\theta}{\mathcal{A}(\theta)\Gamma(\theta)}\beta_i<1,\qquad \text{for } i=1,2,3,4.$$

Proof. It is clear that $\mathcal{W}(\lambda)$, $\mathcal{X}(\lambda)$, $\mathcal{Y}(\lambda)$, and $\mathcal{Z}(\lambda)$ are bounded. The kernels of these functions also satisfy the Lipschitz condition. Therefore, employing the succeeding relation with the application of (3.6), we derived:

$$\left\|\Delta_\upsilon(\lambda)\right\|\leqslant\left\|\mathcal{W}(0)\right\|\left[\frac{1-\theta}{\mathcal{A}(\theta)}\beta_1+\frac{\lambda_{\max}^\theta}{\mathcal{A}(\theta)\Gamma(\theta)}\beta_1\right]^\upsilon,$$

$$\left\|\aleph_\upsilon(\lambda)\right\|\leqslant\left\|\mathcal{X}(0)\right\|\left[\frac{1-\theta}{\mathcal{A}(\theta)}\beta_2+\frac{\lambda_{\max}^\theta}{\mathcal{A}(\theta)\Gamma(\theta)}\beta_2\right]^\upsilon,$$

$$\left\|\Psi_\upsilon(\lambda)\right\|\leqslant\left\|\mathcal{Y}(0)\right\|\left[\frac{1-\theta}{\mathcal{A}(\theta)}\beta_3+\frac{\lambda_{\max}^\theta}{\mathcal{A}(\theta)\Gamma(\theta)}\beta_3\right]^\upsilon,$$

$$\left\|F_\upsilon(\lambda)\right\|\leqslant\left\|\mathcal{Z}(0)\right\|\left[\frac{1-\theta}{\mathcal{A}(\theta)}\beta_4+\frac{\lambda_{\max}^\theta}{\mathcal{A}(\theta)\Gamma(\theta)}\beta_4\right]^\upsilon.$$

Hence, (3.4) is a smooth function and exists.
Let us suppose, that the aforementioned functions represent the model's solutions

$$\begin{aligned}\mathcal{W}(\lambda)-\mathcal{W}(0)&=\mathcal{W}_\upsilon(\lambda)-\Theta_{1(\upsilon)}(\lambda),\\ \mathcal{X}(\lambda)-\mathcal{X}(0)&=\mathcal{X}_\upsilon(\lambda)-\Theta_{2(\upsilon)}(\lambda),\\ \mathcal{Y}(\lambda)-\mathcal{Y}(0)&=\mathcal{Y}_\upsilon(\lambda)-\Theta_{3(\upsilon)}(\lambda),\\ \mathcal{Z}(\lambda)-\mathcal{Z}(0)&=\mathcal{Z}_\upsilon(\lambda)-\Theta_{4(\upsilon)}(\lambda).\end{aligned}$$

The term $\left\|\Theta_\infty(\lambda)\right\|\to 0$ at infinity. To show this, the following is taken

$$\left\|\Theta_{1(\upsilon)}(\lambda)\right\|\leqslant\left\|\frac{1-\theta}{\mathcal{A}(\theta)}\mathcal{K}_1(\lambda,\mathcal{W})-\mathcal{K}_1\left(\lambda,\mathcal{W}_{\upsilon-1}\right)+\frac{\theta}{\mathcal{A}(\theta)\Gamma(\theta)}\int_0^\lambda(\lambda-y)^{\theta-1}(\mathcal{K}_1(\lambda,\mathcal{W})-\mathcal{K}_1\left(\lambda,\mathcal{W}_{\upsilon-1}\right))\mathrm{d}y\right\|,$$

$$\begin{aligned}\left\|\Theta_{1(\upsilon)}(\lambda)\right\|&\leqslant\frac{1-\theta}{\mathcal{A}(\theta)}\left\|\mathcal{K}_1(\lambda,\mathcal{W})-\mathcal{K}_1\left(\lambda,\mathcal{W}_{\upsilon-1}\right)\right\|\\ &\quad+\frac{\theta}{\mathcal{A}(\theta)\Gamma(\theta)}\int_0^\lambda(\lambda-y)^{\theta-1}\left\|\mathcal{K}_1(\lambda,\mathcal{W})-\mathcal{K}_1\left(\lambda,\mathcal{W}_{\upsilon-1}\right)\right\|\mathrm{d}y,\\ &\leqslant\frac{1-\theta}{\mathcal{A}(\theta)}\beta_1\left\|\mathcal{W}-\mathcal{W}_{\upsilon-1}\right\|+\frac{\lambda^\theta}{\mathcal{A}(\theta)\Gamma(\theta)}\beta_1\left\|\mathcal{W}-\mathcal{W}_{\upsilon-1}\right\|.\end{aligned}$$

By recursively repeating the process, we get

$$\left\|\Theta_{1(\upsilon)}(\lambda)\right\|\leqslant\left[\frac{1-\theta}{\mathcal{A}(\theta)}+\frac{\lambda^\theta}{\mathcal{A}(\theta)\Gamma(\theta)}\right]^{\upsilon+1}\beta_1^\upsilon M.$$

So, for $\lambda_{\max}$ we get

$$\left\|\Theta_{1(\upsilon)}(\lambda)\right\|\leqslant\left[\frac{1-\theta}{\mathcal{A}(\theta)}+\frac{\lambda_{\max}^\theta}{\mathcal{A}(\theta)\Gamma(\theta)}\right]^{\upsilon+1}\beta_1^\upsilon M.$$

By taking limit on both sides as $\upsilon\to\infty$, we get $\left\|\Theta_\infty(\lambda)\right\|\to 0$. □

Uniqueness of the solution

The ability to demonstrate the system's uniqueness is an important application. So, via contraction, we suppose that there is another system of solutions to (2.2), $\mathcal{W}_1(\lambda)$, $\mathcal{X}_1(\lambda)$, $\mathcal{Y}_1(\lambda)$ and $\mathcal{Z}_1(\lambda)$. Then

$$\left\|\mathcal{W}(\lambda)-\mathcal{W}_1(\lambda)\right\|\leqslant\frac{1-\theta}{\mathcal{A}(\theta)}\left(\mathcal{K}_1(\lambda,\mathcal{W})-\mathcal{K}_1\left(\lambda,\mathcal{W}_1\right)\right)+\frac{\theta}{\mathcal{A}(\theta)\Gamma(\theta)}\int_0^\lambda(\lambda-y)^{\theta-1}\left(\mathcal{K}_1(\lambda,\mathcal{W})-\mathcal{K}_1\left(\lambda,\mathcal{W}_1\right)\right)dy. \tag{3.7}$$

Now applying the norm to (3.7), we compute

$$\left\|\mathcal{W}(\lambda)-\mathcal{W}_1(\lambda)\right\|\leqslant\frac{1-\theta}{\mathcal{A}(\theta)}\left\|\mathcal{K}_1(\lambda,\mathcal{W})-\mathcal{K}_1\left(\lambda,\mathcal{W}_1\right)\right\|+\frac{\theta}{\mathcal{A}(\theta)\Gamma(\theta)}\int_0^\lambda(\lambda-y)^{\theta-1}\left\|\mathcal{K}_1(\lambda,\mathcal{W})-\mathcal{K}_1\left(\lambda,\mathcal{W}_1\right)\right\|dy.$$

By utilizing the kernel's Lipschitz condition properties, we obtain

$$\left\|\mathcal{W}(\lambda)-\mathcal{W}_1(\lambda)\right\|\leqslant\frac{1-\theta}{\mathcal{A}(\theta)}\left\|\mathcal{W}(\lambda)-\mathcal{W}_1(\lambda)\right\|\beta_1+\frac{\beta_1\lambda^\theta}{\mathcal{A}(\theta)\Gamma(\theta)}\left\|\mathcal{W}(\lambda)-\mathcal{W}_1(\lambda)\right\|,$$

which gives

$$\left\|\mathcal{W}(\lambda)-\mathcal{W}_1(\lambda)\right\|\left(1-\beta_1\frac{1-\theta}{\mathcal{A}(\theta)}+\frac{\lambda^\theta\beta_1}{\mathcal{A}(\theta)\Gamma(\theta)}\right)\leqslant 0,$$
$$\left\|\mathcal{W}(\lambda)-\mathcal{W}_1(\lambda)\right\|=0.$$

Thus, $\mathcal{W}(\lambda)=\mathcal{W}_1(\lambda)$. So the equation has a unique solution. In same manner, one can obtain the same outcomes for other solutions of $\mathcal{X}(\lambda),\mathcal{Y}(\lambda)$ and $\mathcal{Z}(\lambda)$.

4. Numerical scheme

A unique numerical approach for solving several particular problems with non-singular, non-local kernels of fractional derivatives has been proposed by Toufik and Atangana [13]. It is apparent from their study that their technique is not only remarkably precise but also rapidly converges to exact solutions. As other well-known methods such as Eulers, Adams-Bashforth, etc. cannot efficiently and convergently solve the fractional derivatives of non-singular, non-local kernels, this method has the benefit of enhancing the limitations of those methods. By taking the aforementioned non-linear fractional ordinary equation to illustrate their methodology:

$$\begin{cases} {}_0^{ABC}D_\lambda^\theta \varkappa(\lambda)=f(\lambda,\varkappa(\lambda)),\\ \varkappa(0)=\varkappa_0. \end{cases} \tag{4.1}$$

Using the core concepts of fractional calculus, the (4.1) is converted into a fractional integral equation.

$$\varkappa(\lambda)-\varkappa(0)=\frac{1-\theta}{\mathcal{A}(\theta)}f(\lambda,\varkappa(\lambda))+\frac{\theta}{\mathcal{A}(\theta)\Gamma(\theta)}\int_0^\lambda f(y,\varkappa(y))(\lambda-y)^{\theta-1}\,\mathrm{d}y. \tag{4.2}$$

At the point $\lambda=\lambda_{\upsilon+1},\upsilon=0,1,2,\ldots$ the above (4.2) can be written as

$$\varkappa\left(\lambda_{\upsilon+1}\right)-\varkappa(0)=\frac{1-\theta}{\mathcal{A}(\theta)}f\left(\lambda_\upsilon,\varkappa\left(\lambda_\upsilon\right)\right)+\frac{\theta}{\mathcal{A}(\theta)\Gamma(\theta)}\int_0^{\lambda_{\upsilon+1}} f(y,\varkappa(y))\left(\lambda_{\upsilon+1}-y\right)^{\theta-1}\,\mathrm{d}y. \tag{4.3}$$

Let us assume $f(y,\varkappa(y))$ in the interval $\left[\lambda_l,\lambda_{l+1}\right]$ and applying Lagrange interpolation, we get:

$$\begin{aligned} p_l(y)&=f(y,\varkappa(y)),\\ &=\frac{y-\lambda_{l-1}}{\lambda_l-\lambda_{l-1}}f\left(\lambda_l,\varkappa\left(\lambda_l\right)\right)-\frac{y-\lambda_l}{\lambda_l-\lambda_{l-1}}f\left(\lambda_{l-1},\varkappa\left(\lambda_{l-1}\right)\right),\\ &=\frac{f\left(\lambda_l,\varkappa\left(\lambda_l\right)\right)}{h}\left(y-\lambda_{l-1}\right)-\frac{f\left(\lambda_{l-1},\varkappa\left(\lambda_{l-1}\right)\right)}{h}\left(y-\lambda_l\right),\\ &\simeq\frac{f\left(\lambda_l,\varkappa_l\right)}{h}\left(y-\lambda_{l-1}\right)-\frac{f\left(\lambda_{l-1},\varkappa_{l-1}\right)}{h}\left(y-\lambda_l\right). \end{aligned}$$

By assuming (4.3) again for $f(y,\varkappa(y))$ and applying Lagrange interpolation where h is step length, one gets:

$$\begin{aligned}\varkappa_{\upsilon+1}=&\frac{1-\theta}{\mathcal{A}(\theta)}f\left(\lambda_\upsilon,\varkappa\left(\lambda_\upsilon\right)\right)\\ &+\frac{\theta}{\mathcal{A}(\theta)\Gamma(\theta)}\sum_{l=0}^{\upsilon}\left(\frac{f\left(\lambda_l,\varkappa_l\right)}{h}\int_{\lambda_l}^{\lambda_{l+1}}\left(y-\lambda_{l-1}\right)\left(\lambda_{\upsilon+1}-y\right)^{\theta-1}\,\mathrm{d}y\right.\\ &\left.-\frac{f\left(\lambda_{l-1},\varkappa_{l-1}\right)}{h}\int_{\lambda_l}^{\lambda_{l+1}}\left(y-\lambda_l\right)\left(\lambda_{\upsilon+1}-y\right)^{\theta-1}\,\mathrm{d}y\right)+\varkappa_0.\end{aligned} \tag{4.4}$$

Let us take, the following equation without losing its generality

$$P_{\theta,l,1}=\int_{\lambda_l}^{\lambda_{l+1}}\left(y-\lambda_{l-1}\right)\left(\lambda_{\upsilon+1}-y\right)^{\theta-1}dy, \tag{4.5}$$

and

$$P_{\theta,l,2}=\int_{\lambda_l}^{\lambda_{l+1}}\left(y-\lambda_l\right)\left(\lambda_{\upsilon+1}-y\right)^{\theta-1}dy, \tag{4.6}$$

so that

$$P_{\theta,l,1}=h^{\theta+1}\frac{(\upsilon-l+1)^\theta(\upsilon-l+\theta+2)-(\upsilon-l)^\theta(\upsilon-l+2+2\theta)}{\theta(\theta+1)}, \tag{4.7}$$

and

$$P_{\theta,l,2}=h^{\theta+1}\frac{(\upsilon-l+1)^{\theta+1}-(\upsilon-l)^\theta(\upsilon+1-l+\theta)}{\theta(\theta+1)}. \tag{4.8}$$

By integrating (4.6), (4.7), (4.8), and replacing them into (4.4), we compute

$$\begin{cases} \varkappa_{\upsilon+1}=\frac{(1-\theta)}{\mathcal{A}(\theta)}f\left(\lambda_\upsilon,\varkappa\left(\lambda_\upsilon\right)\right)+\frac{\theta}{\mathcal{A}(\theta)}\sum_{l=0}^{\upsilon}\times\left(\frac{h^\theta f\left(\lambda_l,\varkappa_l\right)}{\Gamma(\theta+2)}\right.\\ \left.\left((\upsilon-l+1)^\theta(\upsilon-l+\theta+2)-(\upsilon-l)^\theta(\upsilon-l+2+2\theta)\right)\right)-\frac{\theta}{\mathcal{A}(\theta)}\sum_{l=0}^{\upsilon}\\ \left(\frac{h^\theta f\left(\lambda_{l-1},\varkappa_{l-1}\right)}{\Gamma(\theta+2)}\left((\upsilon-l+1)^{\theta+1}-(\upsilon-l)^\theta(\upsilon+1-l+\theta)\right)\right)+\varkappa(0). \end{cases} \tag{4.9}$$

The fractional derivative of ABC is numerically represented in (4.9). By applying this approach in suggested system (2.3), the following results are obtained:

$$\begin{cases} \mathcal{W}_{\upsilon+1}=\frac{(1-\theta)}{\mathcal{A}(\theta)}\mathcal{K}_1\left(\lambda_n,\mathcal{W}\left(\lambda_n\right)\right)+\frac{\theta}{\mathcal{A}(\theta)}\sum_{l=0}^{\upsilon}\left(\frac{h^\theta\mathcal{K}_1\left(\lambda_l,\mathcal{W}_l\right)}{\Gamma(\theta+2)}\right.\\ \left.\left((\upsilon-l+1)^\theta(\upsilon-l+\theta+2)-(\upsilon-l)^\theta(\upsilon-l+2+2\theta)\right)\right)-\frac{\theta}{\mathcal{A}(\theta)}\\ \sum_{l=0}^{\upsilon}\left(\frac{h^\theta\mathcal{K}_1\left(\lambda_{l-1},\mathcal{W}_{l-1}\right)}{\Gamma(\theta+2)}\left((\upsilon-l+1)^{\theta+1}-(\upsilon-l)^\theta(\upsilon+1-l+\theta)\right)\right)+\mathcal{W}(0), \end{cases}$$

$$\begin{cases} \mathcal{X}_{\upsilon+1}=\frac{(1-\theta)}{\mathcal{A}(\theta)}\mathcal{K}_2\left(\lambda_n,\mathcal{X}\left(\lambda_n\right)\right)+\frac{\theta}{\mathcal{A}(\theta)}\sum_{l=0}^{\upsilon}\left(\frac{h^\theta\mathcal{K}_2\left(\lambda_l,\mathcal{X}_l\right)}{\Gamma(\theta+2)}\right.\\ \left.\left((\upsilon-l+1)^\theta(\upsilon-l+\theta+2)-(\upsilon-l)^\theta(\upsilon-l+2+2\theta)\right)\right)-\frac{\theta}{\mathcal{A}(\theta)}\\ \sum_{l=0}^{\upsilon}\left(\frac{h^\theta\mathcal{K}_2\left(\lambda_{l-1},\mathcal{X}_{l-1}\right)}{\Gamma(\theta+2)}\left((\upsilon-l+1)^{\theta+1}-(\upsilon-l)^\theta(\upsilon+1-l+\theta)\right)\right)+\mathcal{X}(0), \end{cases}$$

$$\begin{cases} \mathcal{Y}_{\upsilon+1}=\frac{(1-\theta)}{\mathcal{A}(\theta)}\mathcal{K}_3\left(\lambda_n,\mathcal{Y}\left(\lambda_n\right)\right)+\frac{\theta}{\mathcal{A}(\theta)}\sum_{l=0}^{\upsilon}\left(\frac{h^\theta\mathcal{K}_3\left(\lambda_l,\mathcal{Y}_l\right)}{\Gamma(\theta+2)}\right.\\ \left.\left((\upsilon-l+1)^\theta(\upsilon-l+\theta+2)-(\upsilon-l)^\theta(\upsilon-l+2+2\theta)\right)\right)-\frac{\theta}{\mathcal{A}(\theta)}\\ \sum_{l=0}^{\upsilon}\left(\frac{h^\theta\mathcal{K}_3\left(\lambda_{l-1},\mathcal{Y}_{l-1}\right)}{\Gamma(\theta+2)}\left((\upsilon-l+1)^{\theta+1}-(\upsilon-l)^\theta(\upsilon+1-l+\theta)\right)\right)+\mathcal{Y}(0), \end{cases}$$

$$\begin{cases} \mathcal{Z}_{\upsilon+1}=\frac{(1-\theta)}{\mathcal{A}(\theta)}\mathcal{K}_4\left(\lambda_n,\mathcal{Z}\left(\lambda_n\right)\right)+\frac{\theta}{\mathcal{A}(\theta)}\sum_{l=0}^{\upsilon}\left(\frac{h^\theta\mathcal{K}_4\left(\lambda_l,\mathcal{Z}_l\right)}{\Gamma(\theta+2)}\right.\\ \left.\left((\upsilon-l+1)^\theta(\upsilon-l+\theta+2)-(\upsilon-l)^\theta(\upsilon-l+2+2\theta)\right)\right)-\frac{\theta}{\mathcal{A}(\theta)}\\ \sum_{l=0}^{\upsilon}\left(\frac{h^\theta\mathcal{K}_4\left(\lambda_{l-1},\mathcal{Z}_{l-1}\right)}{\Gamma(\theta+2)}\left((\upsilon-l+1)^{\theta+1}-(\upsilon-l)^\theta(\upsilon+1-l+\theta)\right)\right)+\mathcal{Z}(0). \end{cases}$$

5. Numerical simulation

Here, we focus on the fractional diabetes mellitus model's numerical solution. To enhance the clarity of our findings, we employ simulations to approximate the solutions for the system (1). For our simulation, we consider specific parameter values and t = 80 days is the total simulation time. The parameters used in the system are: the birth rate is $\gamma = 1$, the death rate of untreated patients with diabetes is $\chi = 0.06654$, the rate of infective contact of susceptible individuals to latent individuals is $\eta = 0.0009$, the rate of death of the treated patients with diabetes is $\mu = 0.09281$, and the latent rate of movement of the individual becoming infected is $\varepsilon\upsilon = 0.88187$, $\varrho = 0.13869$ is the natural death rate. These parameter values contribute to the dynamics of the

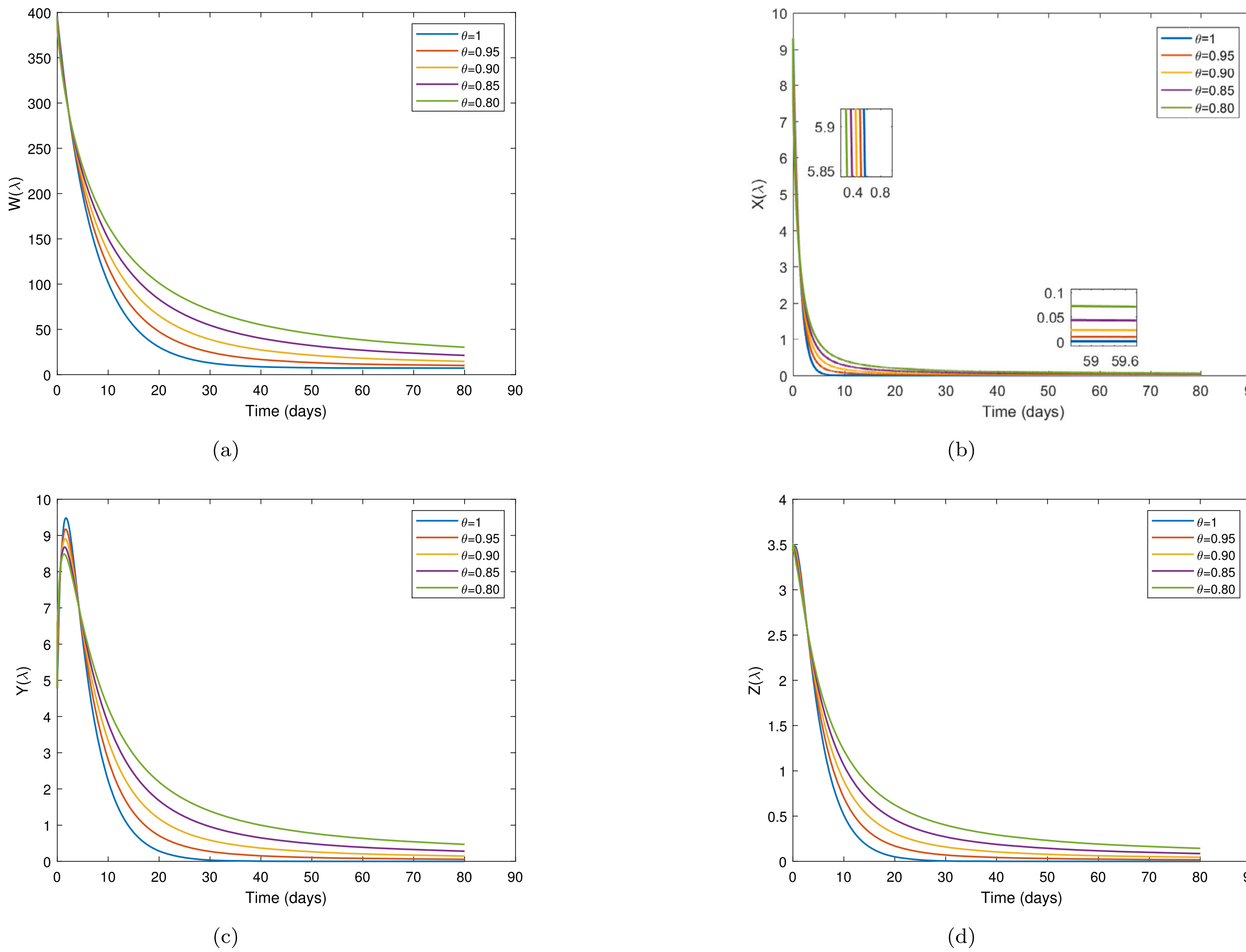

Fig. 2. Simulation of $\mathcal{W}(\lambda)$, $\mathcal{X}(\lambda)$, $\mathcal{Y}(\lambda)$, $\mathcal{Z}(\lambda)$ with $\gamma=1$, $\eta=0.0009$ for different fractional order of θ.

model and allow us to evaluate the behavior of the fractional model (2.3) through numerical simulations. All the simulations are performed and graphically shown using the software MATLAB (R2022a).

Fig. 2 illustrates the dynamic effects observed across a range of values for θ (θ = 0.80, 0.85, 0.90, 0.95, 1). Notably, when considering a fractional order, both the susceptible and infected populations demonstrate an increasing trend. However, as θ approaches 1, there is a significant decrease in these populations. This suggests that a decrease in the θ parameters leads to an increase in both the infected and susceptible populations. The results highlight the influence of θ on the dynamics of the system, emphasizing the importance of considering different values of this parameter in analyzing susceptible and infected populations.

Fig. 3 displays the dynamics of susceptible individuals $\mathcal{W}(\lambda)$, exposed individuals $\mathcal{X}(\lambda)$, infected population $\mathcal{Y}(\lambda)$, and recovered individuals $\mathcal{Z}(\lambda)$ in response to parameter perturbation with η ($\eta = 0.009$), representing the rate of infectious contact between susceptible individuals and the latent population. The figure compares the behavior of the system under both integer and various fractional orders. It provides insights into how the different orders impact the trajectories of the susceptible, exposed, infected, and recovered populations. By examining these variations, we can gain a deeper understanding of the effects of fractional orders on the dynamics of the model and the importance of the parameter η in shaping the population dynamics. Furthermore, the results shown in Fig. 3 offer a foundation for further exploration and refinement of the model, potentially leading to more effective disease control and prevention strategies.

Upon examining the dynamic response presented in Fig. 4, an interesting finding emerged: a higher number of individuals were observed to be living with diabetes mellitus as the values of θ decreased. The parameters $\gamma = 0.04$ and $\eta = 0.009$ were chosen for this analysis, with θ values of 1, 0.90, 0.80, and 0.70. This observation suggests a correlation between the value of θ and the prevalence of diabetes mellitus, indicating that lower values of θ are associated with a greater number of individuals affected by the disease. To minimize the affected class, it is crucial for the fractional order parameter $\theta \to 1$.

As demonstrated in the previous section, the proposed model establishes both the existence and uniqueness of its solution. Similarly, we will substantiate the existence of the solution for the diabetes mellitus model in the context of the ABC operator through numerical experiments. Fig. 5 illustrates the presence of attractors, indicating the enduring coexistence and stability of the entire population over time, irrespective of the fractional orders employed. This observation underscores the importance of fractional calculus in capturing the complex and enduring behavior of the model, demonstrating the inherent stability and resilience of the system. Therefore, the fractional dimension θ assumes a critical role in the simulation experiments of the diabetes mellitus model conducted in this study. In contrast, our study incorporates the ABC fractional order derivative to capture the behavior of the diabetes mellitus model. The simulation outcomes demon-

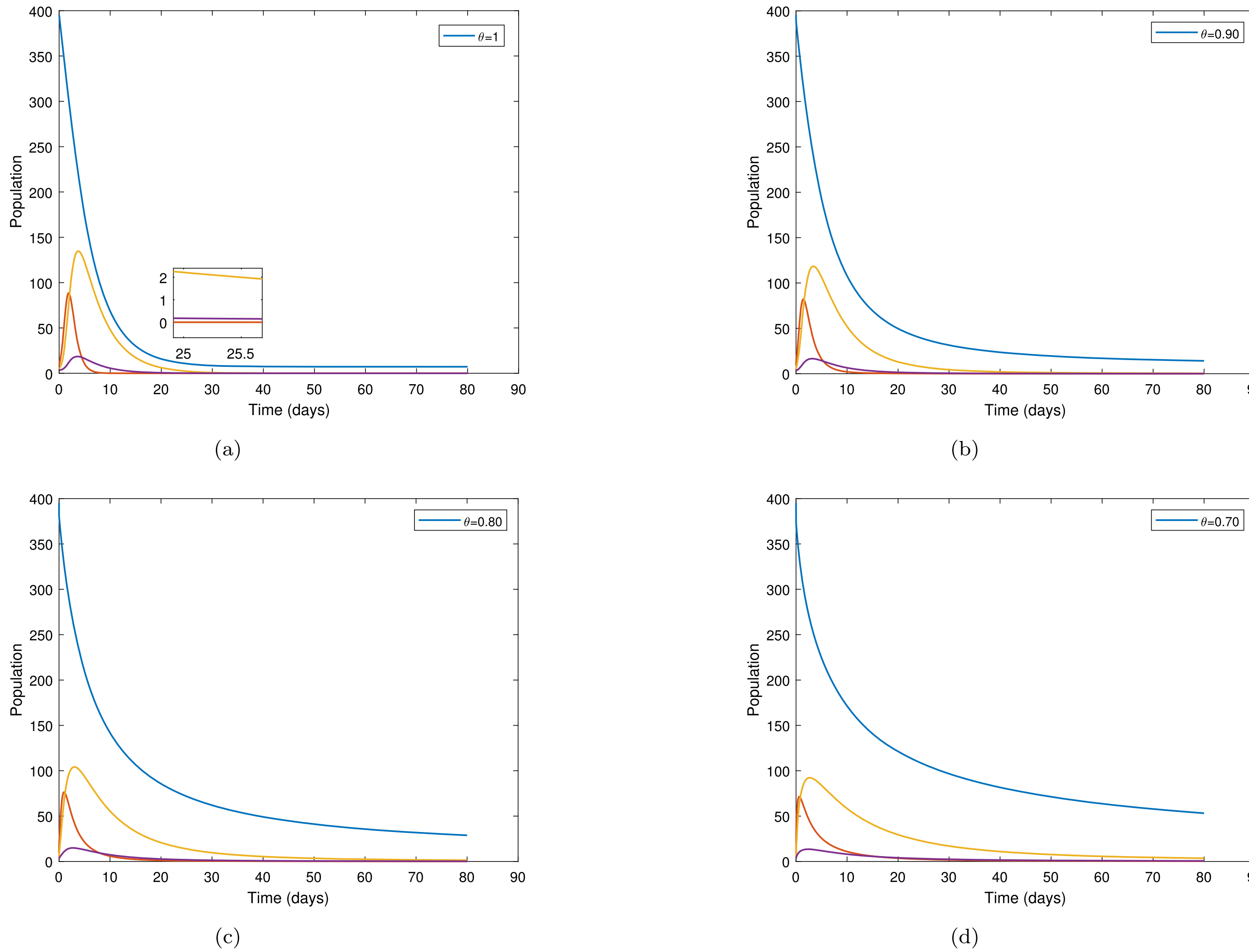

Fig. 3. Simulation outcomes of diabetes mellitus model with different fractional order of θ by setting $\gamma = 1$, $\eta = 0.009$.

strate that even slight adjustments in the derivative order can significantly influence the numerical results. Therefore, when working with real data, it becomes crucial to accurately determine the precise value of the fractional order to attain enhanced precision in the outcomes.

The in-depth qualitative analysis of the findings obtained from the simulation outcomes of the diabetes mellitus model with different fractional orders of θ provides valuable insights into the dynamics of the disease. By considering glucose regulation, insulin sensitivity, oscillatory behavior, long-term stability, and clinical relevance, we can interpret the implications of the results and gain a better understanding of the underlying mechanisms. This knowledge can inform future research, treatment strategies, and potentially contribute to improved management of diabetes mellitus.

6. Conclusion

The application of fractional calculus has been utilized to analyze the mathematical model of diabetes mellitus in this study. Specifically, the model has been modified using the fractional ABC derivative, which incorporates a nonsingular kernel. To discretize the suggested model, an effective numerical approximation approach was developed, which facilitates practical implementation and computer analysis. Through the utilization of fixed point theory, we establish the existence and uniqueness of a system of solutions for the modified diabetes mellitus model under specific conditions. Through simulation experiments, it was observed that as the values of θ decrease and approach 0, there is a notable increase in the number of individuals living with diabetes. Conversely, when θ approaches 1, the number of people living with diabetes tends to decrease. This observation highlights the significant impact of the parameter θ on the prevalence of diabetes, emphasizing the need for effective interventions and strategies to mitigate the rising burden of this disease. In order to obtain a diabetes-free population, it is necessary for the parameter θ to approach unity. This approach allows for a deeper understanding of the dynamics and behavior of diabetes mellitus, contributing to the advancement of research in this field. In the future, the proposed algorithm has the potential for extension to tackle a wide range of biological and physical models. In particular, one can use the proposed model for analyzing the Hepatitis B model, smoking model, alzheimer's disease model, pine wilt disease model of fractional order. To gain deeper insights into the diabetes mellitus model, utilizing non-local and non-singular kernel operators like Caputo-Fabrizio and ABC differential (integral) operators can capture natural phenomena more efficiently than conventional mathematical operators. By employing these fractional operators, it becomes possible to assess their respective strengths and weaknesses in modeling the diabetes mellitus disease. As a result, it would be advantageous for young researchers interested in this field to compare their findings with the outcomes of this study.

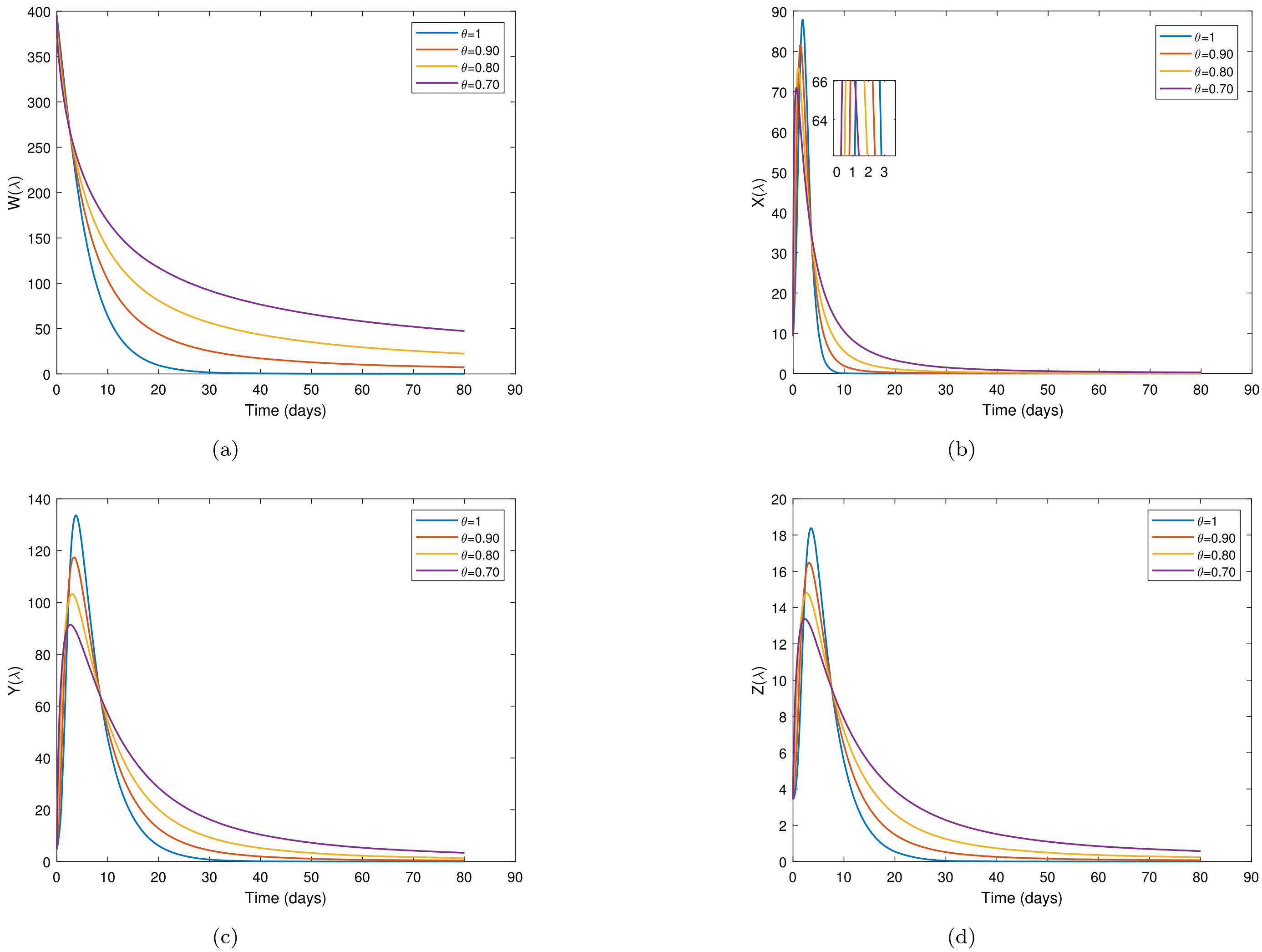

Fig. 4. Simulation of $\mathcal{W}(\lambda)$, $\mathcal{X}(\lambda)$, $\mathcal{Y}(\lambda)$, $\mathcal{Z}(\lambda)$ with $\eta = 0.009$, $\gamma = 0.04$ for different fractional order of θ.

Declaration of competing interest

We declare that there is no conflict of interest among the authors for the publication of this manuscript.

Availability of data and materials

The sharing of data is not relevant to this article since no datasets were produced or examined during the current study.

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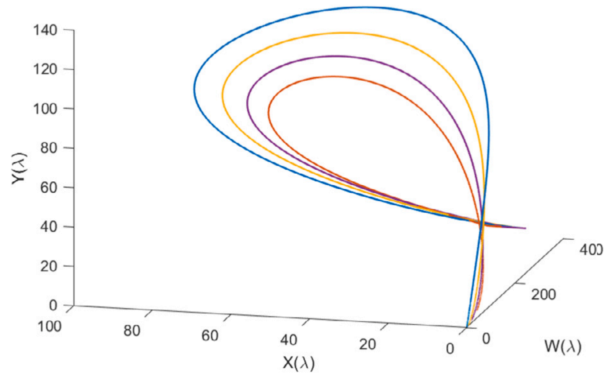
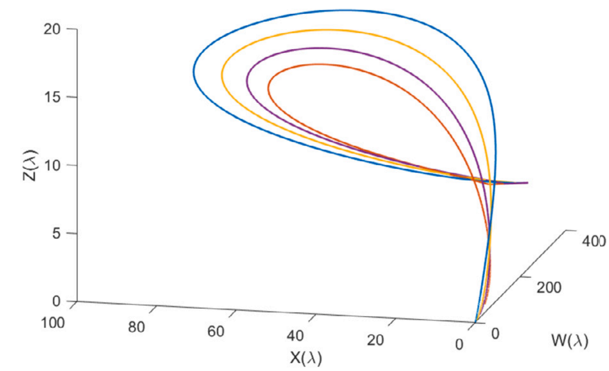


(a)



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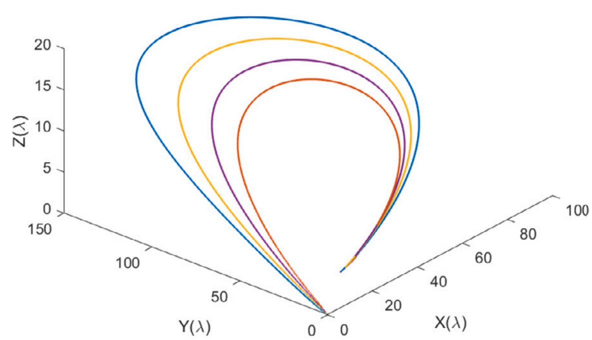

(c)

Fig. 5. The presence of attractors, specifically with the given parameters $\gamma = 0.04$, $\eta = 0.009$, $\varrho = 0.13869$, $\chi = 0.06654$, $\mu = 0.09281$, $\varepsilon\upsilon = 0.88187$, indicates the enduring coexistence and stability of the entire population over time.

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