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Research article

Efficient imputation methods in case of measurement errors

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ABSTRACT

This manuscript develops few efficient difference and ratio kinds of imputations to handle the situation of missing observations given that these observations are polluted by the measurement errors (ME). The mean square errors of the developed imputations are studied to the primary degree approximation by adopting Taylor series expansion. The proposed imputations are equated with the latest existing imputations presented in the literature. The execution of the proposed imputations is assessed by utilizing a broad empirical study utilizing some real and hypothetically created populations. Appropriate remarks are made for sampling respondents regarding practical applications.

1. Introduction

Non-sampling error is an additional source of observational/measurement error that develops whenever the observed values of the sample units differentiate from the real values. Employing unskilled personnel, malfunctioning equipment, and other factors are among the causes of MEs. [1] used conventional ratio estimators to assess the ME's consequence on the population mean. Later, [2] proposed a regression-type procedure for evaluating the population mean under MEs utilizing SRS. [3] used the regression and the ratio estimators to determine the impact of MEs. Whenever the data had been contaminated with ME, [4] evaluated the efficacy of the population total's ratio estimator. Mean, product and ratio estimators have been proposed by [5] under the SRS to assess the impact of MEs. [6] discussed the estimation of population variance under MEs. [7] and [8] suggested some population variance estimation methods utilizing supplementary data in the absence and presence of MEs. [9–11] evaluated the performance of several classes of estimators under the impact of the correlated MEs. Recently, [12] developed few efficient population mean based estimation methods to examine the impact of the correlated MEs.

In survey sampling, most of the survey datasets suffer from the missing data issue, which is a basic and inescapable phenomenon. To overcome this problem, imputation is a well-known method. Many researchers have constructed several imputations to solve the missing data issue. By employing the correlation between supplementary and study variables, the additional data may be utilized at any point of the imputation process, including the survey, design, estimation, or all phases. Observed at random, parameter distribution, and missing at random (MAR) are three fundamental concepts developed by [13]. Furthermore, MAR and missing

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completely at random (MCAR) were discriminated by [14]. Various researchers, namely, [15], [16], [17], [18], [19], [20], [21], [22], [23], [24], [25], [26], [27], and [28] considered MCAR approach and proposed different techniques of imputation and the resulting estimators consisting of supplementary information to handle the missing data issue. But the methods of imputation are seldom utilized for evaluating the population mean under the MEs. This manuscript proposes some difference and ratio kinds of imputations for the population mean estimation under the MCAR approach and the impact of MEs on the resulting estimators is measured.

In Section 2, the formulation of missingness and measurement errors are given along with the notation and symbol for deducing the algebraic expressions of the mean square error (MSE). Section 3 is consisting of the existing imputations in the presence of the MEs. In Section 4, the difference and ratio type methods of imputation are proposed by utilizing the supplementary information. The comparative study of the proposed and existing imputations is performed in Section 5. The execution of the proposed imputations is examined over the existing imputations utilizing an empirical study consisting of the real and artificial data in Section 6, while Section 7 concludes this manuscript.

2. Formulation of missingness and measurement errors

Let $\kappa = (\kappa_1, \kappa_2, ..., \kappa_N)$ be a population of finite size N. To estimate the population mean μ_Y of the study variable y, a sample of size n is taken from κ utilizing simple random sampling without replacement. Let r be the total number of units responding out of n sampled units. Let R be the set of responding units and $\bar{R}$ be the set of non-responding units. For all unit $i \in R$, the amount y_i is determined, but for the unit $i \in \bar{R}$, the amounts are missing and require imputation to accomplish the structure of the sample data set. Let the imputation be performed consisting of the additional auxiliary data, X, so X_i, the i^{th} unit's data of X is useable and positive $\forall\ i \in s$ provided that the data $\mathbf{X_s} = \left\{X_i; i \in s\right\}$ are available.

The missing structure is defined here under as

$$\delta_i = \begin{cases} y_i & \text{when } i \in R \\ \hat{y}_{(.)} & \text{when } i \in \bar{R} \end{cases}$$

where $\hat{y}_{(.)}$ is an imputed value for the i^{th} non-respondent unit on the study variable.

We also take into account the possibility of using ME to measure data values. Let $(y_i, z_i);\ i = 1, 2, ..., n$ be the observed values and (Y_i, X_i) be the true values. Let the observed values be expressed in additive form as $y_i = Y_i + U_i$ and $x_i = X_i + V_i$ given that $U \sim N(0, \sigma_U^2)$ and $V \sim N(0, \sigma_V^2)$. Let the error variables U and V be uncorrelated to each other as well as uncorrelated to other combinations of Y and X, respectively. Let μ_Y, μ_X be the population means and σ_Y^2, σ_X^2 be the population variances of study and auxiliary variables, respectively. In the presence of ME, $s_x^2 = (n-1)^{-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$ and $s_y^2 = (n-1)^{-1}\sum_{i=1}^{n}(y_i - \bar{y})^2$ are not unbiased estimators of the population variances σ_X^2 and σ_Y^2, respectively, *i.e.*, $E(s_x^2) = \sigma_X^2 + \sigma_V^2$ and $E(s_y^2) = \sigma_Y^2 + \sigma_U^2$.

2.1. Notations and symbols

The following notations will be used throughout this study for obtaining the mathematical expressions for MSE. Let $\bar{y}_r = \mu_Y(1 + e_0)$, $\bar{x}_r = \mu_X(1 + e_1)$ and $\bar{x}_n = \mu_X(1 + e_2)$ provided $E(e_i) = 0,\ i = 0, 1, 2$. Up to first order of approximation, we have $E(e_0^2) = f_r\tau_y C_Y^2$, $E(e_1^2) = f_r\tau_x C_X^2$, $E(e_2^2) = f_n\tau_x C_X^2$, $E(e_0e_1) = f_r\rho C_Y C_X$, $E(e_0e_2) = f_n\rho C_Y C_X$, $E(e_1e_2) = f_n\tau_x C_X^2$, where $f_n = \left(\frac{1}{n} - \frac{1}{N}\right)$, $f_r = \left(\frac{1}{r} - \frac{1}{N}\right)$, $f_{rn} = \left(\frac{1}{r} - \frac{1}{n}\right)$. Additionally, the population coefficients of variation for the study and auxiliary variables are $C_Y = S_Y/\mu_Y$ and $C_X = S_X/\mu_X$, respectively. The population correlation coefficient between the variables x and y is $\rho = S_{XY}/S_X S_Y$. Moreover, $\tau_y = (\sigma_Y^2 + \sigma_U^2)/\sigma_Y^2$ and $\tau_x = (\sigma_X^2 + \sigma_V^2)/\sigma_X^2$ are the inverse of the reliability ratio of the study and auxiliary variables.

3. Recap of existing methods of imputation

The present section considers the existing methods of imputation and the corresponding estimators for the estimation of population mean $\bar{Y}$ when the data are contaminated with ME.

3.1. Mean imputation method

The imputation process for the mean approach of imputation is provided by

$$y_{.i_m} = \begin{cases} y_i & \text{when } i \in R \\ \bar{y}_r & \text{when } i \in \bar{R} \end{cases}$$

The resultant estimator is given by

$$t_m = \bar{y}_r$$

For a comprehensive understanding on mean imputations, the authors refer the study of [15].

Taking the auxiliary information into account, we divide the methods of imputation into different schemes as

Scheme I: When μ_X is known and $\bar{x}_n$ is used.
Scheme II: When μ_X is known and $\bar{x}_r$ is used.
Scheme III: When μ_X is not known and $\bar{x}_n$, $\bar{x}_r$ are used.

3.2. Ratio methods of imputation

In the presence of ME, the traditional ratio imputation techniques are provided as

Scheme I

$$y_{.i_{r_1}} = \begin{cases} y_i & \text{when } i \in R \\ \frac{1}{n-r}\left[n\bar{y}_r\left(\frac{\mu_X}{\bar{x}_n}\right) - r\bar{y}_r\right] & \text{when } i \in \bar{R} \end{cases}$$

Scheme II

$$y_{.i_{r_2}} = \begin{cases} y_i & \text{when } i \in R \\ \frac{1}{n-r}\left[n\bar{y}_r\left(\frac{\mu_X}{\bar{x}_r}\right) - r\bar{y}_r\right] & \text{when } i \in \bar{R} \end{cases}$$

Scheme III

$$y_{.i_{r_3}} = \begin{cases} y_i & \text{when } i \in R \\ \frac{1}{n-r}\left[n\bar{y}_r\left(\frac{\bar{x}_n}{\bar{x}_r}\right) - r\bar{y}_r\right] & \text{when } i \in \bar{R} \end{cases}$$

Under the above schemes, the resultant estimators are

$$t_{r_1} = \bar{y}_r\left(\frac{\mu_X}{\bar{x}_n}\right)$$

$$t_{r_2} = \bar{y}_r\left(\frac{\mu_X}{\bar{x}_r}\right)$$

$$t_{r_3} = \bar{y}_r\left(\frac{\bar{x}_n}{\bar{x}_r}\right)$$

3.3. Regression methods of imputation

Influenced by [29], we describe the traditional techniques of imputation for regression in the case of ME as

Scheme I

$$y_{.i_{lr_1}} = \begin{cases} y_i & \text{when } i \in R \\ \bar{y}_r + \frac{n\beta_1}{n-r}(\mu_X - \bar{x}_n) & \text{when } i \in \bar{R} \end{cases}$$

Scheme II

$$y_{.i_{lr_2}} = \begin{cases} y_i & \text{when } i \in R \\ \bar{y}_r + \frac{n\beta_2}{n-r}(\mu_X - \bar{x}_r) & \text{when } i \in \bar{R} \end{cases}$$

Scheme III

$$y_{.i_{lr_3}} = \begin{cases} y_i & \text{when } i \in R \\ \bar{y}_r + \frac{n\beta_3}{n-r}(\bar{x}_n - \bar{x}_r) & \text{when } i \in \bar{R} \end{cases}$$

The resulting estimators are presented as

$$t_{lr_1} = \bar{y}_r + \beta_1(\mu_X - \bar{x}_n)$$

$$t_{lr_2} = \bar{y}_r + \beta_2(\mu_X - \bar{x}_r)$$

$$t_{lr_3} = \bar{y}_r + \beta_3(\bar{x}_n - \bar{x}_r)$$

where the regression coefficients (y on z) are β_i, $i = 1$, 2, and 3.

3.4. Ref. [30] imputation methods

Different logarithmic and sin-type approaches of imputation of population mean μ_Y in the presence of ME were proposed by [30] as

Scheme I

$$y_{.i_{g_1}} = \begin{cases} y_i & \text{when } i \in R \\ \bar{y}_r + \frac{nb_1}{(n-r)} \log\{1 + (\mu_X - \bar{x}_n)\} & \text{when } i \in \bar{R} \end{cases}$$

$$y_{.i_{g_4}} = \begin{cases} y_i & \text{when } i \in R \\ \bar{y}_r + \frac{nb_4}{(n-r)} \sin\left(1 - \frac{\bar{x}_n}{\mu_X}\right) & \text{when } i \in \bar{R} \end{cases}$$

Scheme II

$$y_{.i_{g_2}} = \begin{cases} y_i & \text{when } i \in R \\ \bar{y}_r + \frac{nb_2}{(n-r)} \log\{1 + (\mu_X - \bar{x}_r)\} & \text{when } i \in \bar{R} \end{cases}$$

$$y_{.i_{g_5}} = \begin{cases} y_i & \text{when } i \in R \\ \bar{y}_r + \frac{nb_5}{(n-r)} \sin\left(1 - \frac{\bar{x}_r}{\mu_X}\right) & \text{when } i \in \bar{R} \end{cases}$$

Scheme III

$$y_{.i_{g_3}} = \begin{cases} y_i & \text{when } i \in R \\ \bar{y}_r + \frac{nb_3}{(n-r)} \log\{1 + (\bar{x}_n - \bar{x}_r)\} & \text{when } i \in \bar{R} \end{cases}$$

$$y_{.i_{g_6}} = \begin{cases} y_i & \text{when } i \in R \\ \bar{y}_r + \frac{nb_6}{(n-r)} \sin\left(1 - \frac{\bar{x}_n}{\bar{x}_r}\right) & \text{when } i \in \bar{R} \end{cases}$$

The resulting estimators are provided by

$$t_{g_1} = \bar{y}_r + b_1 \log\{1 + (\mu_X - \bar{x}_n)\}$$

$$t_{g_2} = \bar{y}_r + b_2 \log\{1 + (\mu_X - \bar{x}_r)\}$$

$$t_{g_3} = \bar{y}_r + b_3 \log\{1 + (\bar{x}_n - \bar{x}_r)\}$$

$$t_{g_4} = \bar{y}_r + b_4 \sin\left(1 - \frac{\bar{x}_n}{\mu_X}\right)$$

$$t_{g_5} = \bar{y}_r + b_5 \sin\left(1 - \frac{\bar{x}_r}{\mu_X}\right)$$

$$t_{g_6} = \bar{y}_r + b_6 \sin\left(1 - \frac{\bar{x}_n}{\bar{x}_r}\right)$$

where the scalars $b_i,\ i = 1, 2, ..., 6$ are to be found by minimizing the corresponding MSE equations. Under the corresponding schemes, the minimum MSE of the consequent estimators $t_{g_i},\ i = 1, 2, ..., 6$ achieves the minimum MSE of the regression estimator.

The MSE of the resultant estimators of the existing methods of imputation discussed in this section is given in Appendix A.

4. Proposed methods of imputation

The imputation methods and the resultant estimators suggested by [30] do not compete with the regression methods of imputation and the resultant estimators (best linear unbiased estimators) envisaged on the lines of [29]. Taking this issue into consideration, we propose some efficient difference and ratio type methods of imputation for the estimation of population mean μ_Y under SRS when the data are contaminated with ME as

Scheme I

$$y_{.i_1} = \begin{cases} \alpha_1 y_i & \text{when } i \in R \\ \alpha_1 \bar{y}_r + \frac{n\theta_1}{n-r}(\bar{x}_n - \mu_X) & \text{when } i \in \bar{R} \end{cases}$$

$$y_{.i_4} = \begin{cases} y_i & \text{when } i \in R \\ \frac{1}{n-r}\left\{ n\alpha_4 \bar{y}_r \left(\frac{\mu_X}{\bar{x}_n}\right)^{\theta_4} - r\bar{y}_r \right\} & \text{when } i \in \bar{R} \end{cases}$$

$$y_{.i_7} = \begin{cases} y_i & \text{when } i \in R \\ \frac{1}{n-r}\left\{ n\alpha_7 \bar{y}_r \left(\frac{\mu_X}{\mu_X + \theta_7(\bar{x}_n - \mu_X)}\right) - r\bar{y}_r \right\} & \text{when } i \in \bar{R} \end{cases}$$

Scheme II

$$y_{.i_2} = \begin{cases} \alpha_2 y_i & \text{when } i \in R \\ \alpha_2 \bar{y}_r + \frac{n\theta_2}{n-r}(\bar{x}_r - \mu_X) & \text{when } i \in \bar{R} \end{cases}$$

$$y_{.i_5} = \begin{cases} y_i & \text{when } i \in R \\ \frac{1}{n-r}\left\{ n\alpha_5 \bar{y}_r \left(\frac{\mu_X}{\bar{x}_r}\right)^{\theta_5} - r\bar{y}_r \right\} & \text{when } i \in \bar{R} \end{cases}$$

$$y_{.i_8} = \begin{cases} y_i & \text{when } i \in R \\ \frac{1}{n-r}\left[n\alpha_8 \bar{y}_r \left\{ \frac{\mu_X}{\mu_X + \theta_8(\bar{x}_r - \mu_X)} \right\} - r\bar{y}_r \right] & \text{when } i \in \bar{R} \end{cases}$$

Scheme III

$$y_{.i_3} = \begin{cases} \alpha_3 y_i & \text{when } i \in R \\ \alpha_3 \bar{y}_r + \frac{n\theta_3}{n-r}(\bar{x}_r - \bar{x}_n) & \text{when } i \in \bar{R} \end{cases}$$

$$y_{.i_6} = \begin{cases} y_i & \text{when } i \in R \\ \frac{1}{n-r}\left\{ n\alpha_6 \bar{y}_r \left(\frac{\bar{x}_n}{\bar{x}_r}\right)^{\theta_6} - r\bar{y}_r \right\} & \text{when } i \in \bar{R} \end{cases}$$

$$y_{.i_9} = \begin{cases} y_i & \text{when } i \in R \\ \frac{1}{n-r}\left[n\alpha_9 \bar{y}_r \left\{ \frac{\bar{x}_n}{\bar{x}_n + \theta_9(\bar{x}_r - \bar{x}_n)} \right\} - r\bar{y}_r \right] & \text{when } i \in \bar{R} \end{cases}$$

The resulting estimators for the aforementioned approaches are provided by

$$T_1 = \alpha_1 \bar{y}_r + \theta_1(\bar{x}_n - \mu_X)$$
$$T_2 = \alpha_2 \bar{y}_r + \theta_2(\bar{x}_r - \mu_X)$$
$$T_3 = \alpha_3 \bar{y}_r + \theta_3(\bar{x}_r - \bar{x}_n)$$
$$T_4 = \alpha_4 \bar{y}_r \left(\frac{\mu_X}{\bar{x}_n}\right)^{\theta_4}$$
$$T_5 = \alpha_5 \bar{y}_r \left(\frac{\mu_X}{\bar{x}_r}\right)^{\theta_5}$$
$$T_6 = \alpha_6 \bar{y}_r \left(\frac{\bar{x}_n}{\bar{x}_r}\right)^{\theta_6}$$
$$T_7 = \alpha_7 \bar{y}_r \left\{ \frac{\mu_X}{\mu_X + \theta_7(\bar{x}_n - \mu_X)} \right\}$$
$$T_8 = \alpha_8 \bar{y}_r \left\{ \frac{\mu_X}{\mu_X + \theta_8(\bar{x}_r - \mu_X)} \right\}$$
$$T_9 = \alpha_9 \bar{y}_r \left\{ \frac{\bar{x}_n}{\bar{x}_n + \theta_9(\bar{x}_r - \bar{x}_n)} \right\}$$

where α_j and θ_j; $j = 1, 2, ..., 9$ are suitably chosen scalars.

Theorem 4.1. *The minimum MSE of the resultant difference type estimators $T_j,\ j = 1, 2, 3$ is given by*

$$minMSE(T_j) = \mu_Y^2(1 - \alpha_{j(opt)}) = \mu_Y^2\left(1 - \frac{Q_j^2}{P_j}\right)$$

Proof. Taking the notations given in the previous section, we can write the estimator T_1 as

$$T_1 - \mu_Y = (\alpha_1 - 1)\mu_Y + \alpha_1 \mu_Y e_0 + \theta_1 \mu_X e_1$$

Squaring and taking expectation both sides of the above equation, we get the MSE of estimator T_1 as

$$MSE(T_1) = (\alpha_1 - 1)^2 \mu_Y^2 + \alpha_1^2 \mu_Y^2 f_r \tau_y C_Y^2 + \theta_1^2 \mu_X^2 f_r \tau_x C_X^2 + 2\alpha_1 \theta_1 \mu_Y \mu_X f_r \rho C_Y C_X$$

By minimizing the $MSE(T_1)$ w.r.t. α_1 and θ_1, we get

$$\alpha_{1(opt)} = \frac{1}{\left(1 + f_r\tau_y C_Y^2 - f_r\rho^2 \frac{C_Y^2}{\tau_x}\right)} = \frac{Q_1}{P_1}\ (say)$$

$$\theta_{1(opt)} = -\frac{\mu_Y}{\mu_X}\rho\left(\frac{C_Y}{\tau_x C_X}\right)\alpha_{1(opt)}$$

Putting $\alpha_{1(opt)}$ and $\theta_{1(opt)}$ in $MSE(T_1)$, we get

$$minMSE(T_1) = \mu_Y^2(1 - \alpha_{1(opt)}) = \mu_Y^2\left(1 - \frac{Q_1^2}{P_1}\right)$$

The MSE expressions of other estimators can be obtained in similar lines.

The optimum values of other scalars are provided here under for quick review.

$$\alpha_{2(opt)} = \frac{1}{\left(1 + f_r\tau_y C_Y^2 - f_n\rho^2 \frac{C_Y^2}{\tau_x}\right)}$$

$$\theta_{2(opt)} = -\frac{\mu_Y}{\mu_X}\rho\left(\frac{C_Y}{\tau_x C_X}\right)\alpha_{2(opt)}$$

$$\alpha_{3(opt)} = \frac{1}{\left(1 + f_r\tau_y C_Y^2 - f_{rn}\rho^2 \frac{C_Y^2}{\tau_x}\right)}$$

$$\theta_{3(opt)} = -\frac{\mu_Y}{\mu_X}\rho\left(\frac{C_Y}{\tau_x C_X}\right)\alpha_{3(opt)} \quad \square$$

Theorem 4.2. *The minimum MSE of the resultant ratio type estimators $T_j,\ j = 4, 5, ..., 9$ is expressed by*

$$minMSE(T_j) = \mu_Y^2\left(1 - \frac{Q_j^2}{P_j}\right) \tag{4.1}$$

Proof. We express the estimator T_4 in terms of errors as

$$T_4 - \mu_Y = \mu_Y\left[\alpha_4\left\{1 + e_0 - \theta_4 e_1 - \theta_4 e_0 e_1 + \frac{\theta_4(\theta_4+1)}{2}\right\} - 1\right]$$

Squaring and taking expectation both the sides of above equation, we get the MSE of estimator T_4 as

$$MSE(T_4) = \mu_Y^2\left[\begin{array}{l} 1 + \alpha_4^2\left\{1 + f_r\tau_y C_Y^2 + \theta_4(2\theta_4+1)f_r\tau_x C_X^2 - 4\theta_4 f_r\rho C_Y C_X\right\} \\ -2\alpha_4\left\{1 - \theta_4 f_r\rho C_Y C_X + \frac{\theta_4(\theta_4+1)}{2} f_r\tau_x C_X^2\right\}\end{array}\right]$$

which can be written as

$$MSE(T_4) = \mu_Y^2\left(1 + \alpha_4^2 P_4 - 2\alpha_4 Q_4\right)$$

By minimizing the $MSE(T_4)$ w.r.t. α_4, we obtain

$$\alpha_{4(opt)} = \frac{Q_4}{P_4}$$

Putting $\alpha_{4(opt)}$ in the $MSE(T_4)$, we obtain

$$minMSE(T_4) = \mu_Y^2\left(1 - \frac{Q_4^2}{P_4}\right)$$

Similarly, the derivations of other estimators $T_j,\ j = 5, 6, ..., 9$ can be obtained. In usual, we can write

$$MSE(T_j) = \mu_Y^2\left(1 + \alpha_j^2 P_j - 2\alpha_j Q_j\right)$$

It is worth mentioning that the simultaneous optimization of α_j and θ_j of the MSE equation is not obtainable. Therefore, we consider optimum values of $\theta_j = \theta_{j(opt)}$ when $\alpha_j = 1$ which is used in $\alpha_j = \alpha_{j(opt)}$ to get (4.1).

The optimum values of scalars are provided below:

$$\alpha_{i(opt)} = \frac{Q_i}{P_i},\ i = 4, 5, ..., 9$$

where $P_4 = 1 + f_r\tau_y C_Y^2 + \theta_4(2\theta_4 + 1)f_r\tau_x C_X^2 - 4\theta_4 f_r\rho C_Y C_X$; $Q_4 = 1 - \theta_4 f_r\rho C_Y C_X + \frac{\theta_4(\theta_4+1)}{2} f_r\tau_x C_X^2$; $P_5 = 1 + f_r\tau_y C_Y^2 + \theta_5(2\theta_5 + 1)f_n\tau_x C_X^2 - 4\theta_5 f_n\rho C_Y C_X$; $Q_5 = 1 - \theta_5 f_n\rho C_Y C_X + \frac{\theta_5(\theta_5+1)}{2} f_n\tau_x C_X^2$; $P_6 = 1 + f_r\tau_y C_Y^2 + \theta_6(2\theta_6 + 1)f_{rn}\tau_x C_X^2 - 4\theta_6 f_{rn}\rho C_Y C_X$; $Q_6 = 1 - \theta_6 f_r\rho C_Y C_X + \frac{\theta_6(\theta_6+1)}{2} f_{rn}\tau_x C_X^2$; $P_7 = 1 + f_r\tau_y C_Y^2 + 3\theta_7^2 f_r\tau_x C_X^2 - 4\theta_7 f_r\rho C_Y C_X$; $Q_7 = 1 + \theta_7^2 f_r\tau_x C_X^2 - \theta_7 f_r\rho C_Y C_X$; $P_8 = 1 + f_r\tau_y C_Y^2 + 3\theta_8^2 f_n\tau_x C_X^2 - 4\theta_8 f_n\rho C_Y C_X$; $Q_8 = 1 + \theta_8^2 f_n\tau_x C_X^2 - \theta_8 f_n\rho C_Y C_X$; $P_9 = 1 + f_r\tau_y C_Y^2 + 3\theta_9^2 f_{rn}\tau_x C_X^2 - 4\theta_9 f_{rn}\rho C_Y C_X$ and $Q_9 = 1 + \theta_9^2 f_r\tau_y C_Y^2 - \theta_9 f_{rn}\rho C_Y C_X$.

The optimum values of $\theta_i,\ i = 4, 5, \ldots, 9$ are given as

$$\theta_{i(opt)} = \rho\frac{C_Y}{\tau_x C_X}$$

which are obtained by putting $\alpha_i = 1$ in the particular estimators and minimizing their MSE expressions w.r.t. scalars. □

5. Comparative study

In this section, we compare the proposed methods of imputation with the existing methods of imputation under schemes I, II and III.

Lemma 5.1. *The proposed imputations $y_{.i_j}$, $j = 1, 2, \ldots, 9$ surpass the mean imputation $y_{.i_m}$ in schemes I, II and III, if*

$$MSE(T_j) < MSE(T_m) \implies \frac{Q_i^2}{P_i} > 1 - f_r\tau_y C_Y^2$$

Lemma 5.2. *(i). The proposed imputations $y_{.i_j}$, $j = 1, 4, 7$ surpass the ratio imputation $y_{.i_{r_1}}$ in scheme I, if*

$$MSE(T_j) < MSE(T_r) \implies \frac{Q_1^2}{P_1} > 1 - f_r\tau_y C_Y^2 - f_n\tau_x C_X^2 - 2f_n\rho C_X C_Y$$

(ii). The proposed imputations $y_{.i_j}$, $j = 2, 5, 8$ surpass the ratio imputation $y_{.i_{r_2}}$ in scheme II, if

$$MSE(T_j) < MSE(T_r) \implies \frac{Q_2^2}{P_2} > 1 - f_r\tau_y C_Y^2 + f_r\tau_x C_X^2 - 2f_r\rho C_X C_Y$$

(iii). The proposed imputations $y_{.i_j}$, $j = 3, 6, 9$ surpass the ratio imputation $y_{.i_{r_3}}$ in scheme III, if

$$MSE(T_j) < MSE(T_r) \implies \frac{Q_3^2}{P_3} > 1 - f_n\tau_y C_Y^2 - f_{rn}\tau_y C_Y^2 + f_{rn}\tau_x C_X^2 - 2f_{rn}\rho C_X C_Y$$

Lemma 5.3. *(i). The proposed imputations $y_{.i_j}$, $j = 1, 4, 7$ surpass the regression imputation $y_{.i_{lr_1}}$ in scheme I, if*

$$MSE(T_j) < MSE(T_{lr_1}) \implies \frac{Q_1^2}{P_1} > 1 - f_r\tau_y C_Y^2 + f_n\tau_y C_Y^2\rho^2$$

(ii). The proposed imputations $y_{.i_j}$, $j = 2, 5, 8$ surpass the regression imputation $y_{.i_{lr_2}}$ in scheme II, if

$$MSE(T_j) < MSE(T_{lr_2}) \implies \frac{Q_2^2}{P_2} > 1 - f_r\tau_y C_Y^2(1 - \rho^2)$$

(iii). The proposed imputations $y_{.i_j}$, $j = 3, 6, 9$ surpass the regression imputation $y_{.i_{lr_3}}$ in scheme III, if

$$MSE(T_j) < MSE(T_{lr_3}) \implies \frac{Q_3^2}{P_3} > 1 - f_n\tau_y C_Y^2 - f_{rn}\tau_y C_Y^2(1 - \rho^2)$$

Lemma 5.4. *(i). The proposed imputations $y_{.i_j}$, $j = 1, 4, 7$ surpass ref. [30] imputations $y_{.i_{g_j}}$, $j = 1, 4$ in scheme I, if*

$$MSE(T_j) < MSE(T_{g_i}) \implies \frac{Q_1^2}{P_1} > 1 - f_r\tau_y C_Y^2 + f_n\tau_y C_Y^2\rho^2$$

(ii). The proposed imputations $y_{.i_j}$, $j = 2, 5, 8$ surpass ref. [30] imputations $y_{.i_{g_j}}$, $j = 2, 5$ in scheme II, if

$$MSE(T_j) < MSE(T_{g_i}) \implies \frac{Q_2^2}{P_2} > 1 - f_r\tau_y C_Y^2(1 - \rho^2)$$

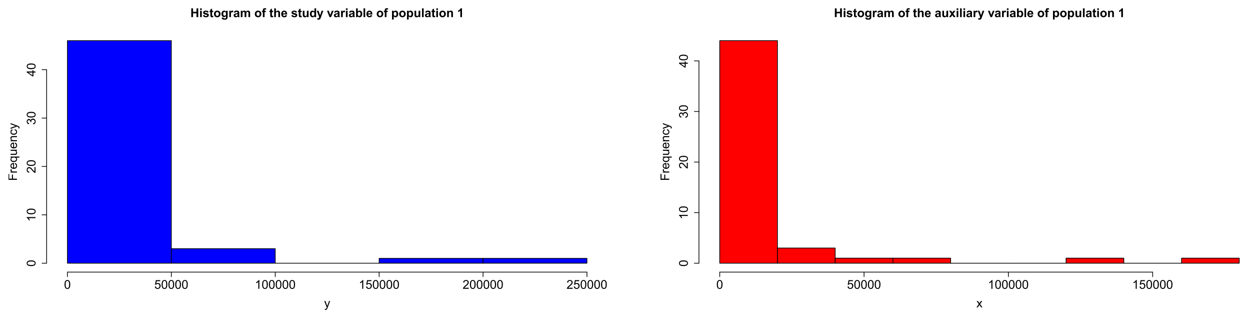

Fig. 1. The histograms for the study and auxiliary variables of population 1.

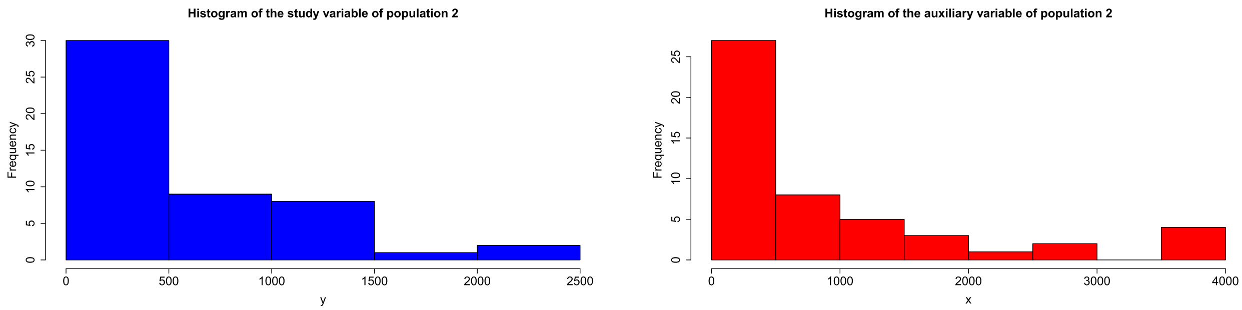

Fig. 2. The histograms for the study and auxiliary variables of population 2.

(iii). The proposed imputations $y_{.i_j}$, $j=3,6,9$ surpass ref. [30] imputations $y_{.i_{g_j}}$, $j=3,6$ in scheme III, if

$$MSE(T_j) < MSE(T_{g_i}) \implies \frac{Q_3^2}{P_3} > 1 - f_n \tau_y C_Y^2 - f_{rn} \tau_y C_Y^2 (1-\rho^2)$$

The proposed imputations dominate the existing imputation under the above lemmas.

6. Empirical study

To strengthen the theoretical results, an empirical study is performed in two subsections, namely, a numerical study based on real populations and a simulation studies based on a hypothetical population.

6.1. Numerical study

In this section, we have performed a numerical study using two real populations.

Population 1: Source: Ref. [31], page number-1129.
Y_i = true number of immigrants admitted in USA in 1996, y_i = measured number of immigrants admitted in USA in 1996, X_i = true number of immigrants admitted in USA in 1995, and x_i = measured number of immigrants admitted in USA in 1995.
$N=51$, $n=20$, $r=17$, $\mu_y = 17702.76$, $\mu_x = 13903.24$, $\sigma_y = 37656.45$, $\sigma_x = 30260.87$, $\sigma_u = 753.12$, $\sigma_v = 605.21$, and $\rho = 0.99$.

Population 2: Source: Ref. [31], page number-1111.
Y_i = true amount (in \$000) of real estate farm loans in various states in 1997, y_i = observed amount (in \$000) of real estate farm loans in various states in 1997, X_i = true amount (in \$000) of nonreal estate farm loans in various states in 1997, and x_i = measured amount (in \$000) of nonreal estate farm loans in various states in 1997. The descriptive values of the population are given as follows.
$N=50$, $n=18$, $r=14$, $\mu_y = 533.43$, $\mu_x = 878.16$, $\sigma_y = 579.60$, $\sigma_x = 1084.67$, $\sigma_u = 11.59$, $\sigma_v = 21.69$, and $\rho = 0.79$.

The above two populations are shown through the histograms in Fig. 1 and Fig. 2, respectively.

Percent relative efficiencies (PREs) are computed using the following formula:

$$PRE = \frac{MSE(t_m)}{MSE(T^*)} \times 100$$

where $T^* = t_m,\ t_{r_1},\ t_{r_2},\ t_{r_3},\ t_{lr_1},\ t_{lr_2},\ t_{lr_3},\ t_{g_1},\ t_{g_2}, \ldots, t_{g_6},\ T_1,\ T_2, \ldots, T_9$.

The findings of the numerical study are reported in Table 1 which exhibits that the proposed estimators repress the currently used estimators in both populations. The results of Table 1 also show that the proposed estimator T_4, T_5, and T_6 are the best in schemes I, II, and III among the proposed estimators in both populations. These results are further generalized by an extensive simulation study.

Table 1
MSE and PRE of different estimators for real populations.

	Population 1		Population 2	
Estimators	MSE	PRE	MSE	PRE
t_m	70928771	100	18671	100
Scheme I				
t_{r_1}	13040783	543	11179	167
t_{lr_1}	13004796	545	9767	191
t_{g_1}	13004796	545	9767	191
T_1	12486633	568	9443	197
T_4	12438260	570	9331	200
T_7	12486633	568	9443	197
Scheme II				
t_{r_2}	676052	10491	8453	220
t_{lr_2}	629618	11265	6412	291
t_{g_2}	629618	11265	6412	291
T_2	628356	11287	6270	297
T_5	623151	11382	6152	303
T_8	628356	11287	6270	297
Scheme III				
t_{r_3}	43265678	163	14558	128
t_{lr_3}	43255231	164	13928	134
t_{g_3}	43255231	164	13928	134
T_3	38009048	186	13278	140
T_6	37970675	187	13212	141
T_9	38009048	186	13278	140

6.2. Simulation study

To have a better understanding about the efficiency of the proposed imputations in comparison with the existing imputations, a simulation study is accomplished. The R language is used to draw a normal population of $N = 1000$ size from a multivariate normal distribution based on mean vector $(\mu_Y, \mu_X, 0, 0)$ and covariance matrix

$$\begin{pmatrix} \sigma_Y^2 & \rho\sigma_X\sigma_Y & 0 & 0 \\ \rho\sigma_X\sigma_Y & \sigma_X^2 & 0 & 0 \\ 0 & 0 & \sigma_U^2 & 0 \\ 0 & 0 & 0 & \sigma_V^2 \end{pmatrix}$$

such that $\mu_Y = 15$, $\mu_X = 10$, $\sigma_Y = 30, 35$, $\sigma_X = 20, 25$, $\sigma_U = 15, 20$, $\sigma_V = 20, 25$, $r = 370, 380$ and $\rho = 0.6, 0.7, 0.8, 0.9$. Using 50,000 iterations and taking imputation technique into account to tabulate the MSE and PRE of the resultant estimators. The following points describe the simulation study.

(i). Select a size n random sample s from the size N population.
(ii). From sample s, drop down randomly (n-r) units every time.
(iii). The drop-down units are imputed utilizing proposed methods of imputation.
(iv). The statistics required are calculated.
(v). The above points are iterated 50,000 times.

The R code is available upon request from the first author. The formulae adopted to tabulate the MSE and PRE are provided hereunder.

$$MSE(T_i) = \frac{1}{50,000}\sum_{i=1}^{50,000}(T_i - \mu_Y)^2$$

$$PRE = \frac{MSE(t_m)}{MSE(T_i)} \times 100$$

The findings of the simulation study are disclosed in Tables 2–9 with MSE and PRE for various combinations of σ_Y, σ_X, σ_U and σ_V. To examine how the suggested imputations behave, simulation findings are generated for a set of meticulously analysed response units ($r = 370, 380$) and correlation coefficients ($\rho = 0.6, 0.7, 0.8, 0.9$). From simulation findings disclosed in Table 2 to Table 9, the following observations are pointed out:

(i). The proposed methods of imputation outperform the mean, ratio, and regression methods of imputation as well as ref. [30] methods of imputation with lower MSE and higher PRE for various correlation coefficients and response rates, according to Table 2 based on $\sigma_Y = 30$, $\sigma_X = 20$, $\sigma_U = 15$ and $\sigma_V = 20$.

Table 2
MSE and PRE of estimators for $(\sigma_Y, \sigma_X) = (30, 20)$ and $(\sigma_U, \sigma_V) = (15, 20)$.

Response	r = 370								r = 380							
ρ	0.6		0.7		0.8		0.9		0.6		0.7		0.8		0.9	
Estimators	MSE	PRE	MSE	PRE	MSE	PRE	MSE	PRE	MSE	PRE	MSE	PRE	MSE	PRE	MSE	PRE
t_m	2.76	100	2.76	100	2.72	100	2.76	100	2.76	100	2.76	100	2.72	100	2.76	100
Scheme I																
t_{r_1}	2.92	94.66	2.65	104.44	2.50	108.72	2.12	129.73	2.84	97.27	2.57	107.63	2.43	112.14	2.05	134.69
t_{lr_1}	1.65	167.27	1.56	176.01	1.44	188.88	1.36	202.94	1.57	175.79	1.49	185.23	1.35	201.48	1.27	214.17
t_{g_1}	1.65	167.27	1.56	176.01	1.44	188.88	1.36	202.94	1.57	175.79	1.49	185.23	1.35	201.48	1.27	214.17
T_1	1.62	170.57	1.53	180.12	1.40	194.05	1.33	208.16	1.54	179.13	1.46	189.70	1.33	205.23	1.25	221.09
T_4	1.62	170.97	1.53	180.59	1.39	194.63	1.32	208.76	1.54	179.55	1.45	190.19	1.32	205.84	1.24	221.73
T_7	1.62	170.57	1.53	180.12	1.40	194.05	1.33	208.16	1.54	179.13	1.46	189.70	1.33	205.23	1.25	221.09
Scheme II																
t_{r_2}	3.06	90.32	2.75	100.52	2.59	105.04	2.16	127.71	2.93	94.25	2.63	104.89	2.48	109.56	2.07	133.28
t_{lr_2}	1.60	172.50	1.51	182.78	1.37	198.54	1.28	215.262	1.55	178.06	1.46	189.404	1.31	207.63	1.25	220.80
t_{g_2}	1.60	172.50	1.51	182.78	1.37	198.54	1.28	215.262	1.55	178.06	1.46	189.404	1.31	207.63	1.25	220.80
T_2	1.58	174.17	1.49	185.59	1.34	202.62	1.25	220.32	1.52	181.69	1.43	193.60	1.29	211.37	1.20	228.74
T_5	1.58	174.64	1.49	186.14	1.34	203.31	1.25	221.05	1.52	182.16	1.42	194.15	1.28	212.06	1.19	229.84
T_8	1.29	174.17	1.49	185.59	1.34	202.62	1.25	220.32	1.52	181.69	1.43	193.60	1.29	211.37	1.20	229.84
Scheme III																
t_{r_3}	2.02	136.65	1.99	139.19	1.94	140.19	1.91	144.26	1.89	145.86	1.87	147.74	1.83	148.45	1.82	151.39
t_{lr_3}	1.90	145.26	1.84	150.48	1.80	151.11	1.82	151.64	1.80	153.33	1.78	155.05	1.75	155.42	1.76	156.81
t_{g_3}	1.90	145.26	1.84	150.48	1.80	151.11	1.82	151.64	1.80	153.33	1.78	155.05	1.75	155.42	1.76	156.81
T_3	1.85	150.64	1.83	151.59	1.78	152.79	1.79	153.94	1.77	156.23	1.76	156.89	1.74	157.72	1.74	158.52
T_6	1.83	150.69	1.82	151.65	1.78	152.85	1.79	153.99	1.77	156.26	1.76	156.93	1.72	157.75	1.74	158.55
T_9	1.84	150.64	1.82	151.59	1.78	152.79	1.79	153.94	1.77	156.23	1.76	156.89	1.73	157.72	1.74	158.52

Table 3
MSE and PRE of estimators for $(\sigma_Y, \sigma_X) = (30, 20)$ and $(\sigma_U, \sigma_V) = (15, 25)$.

Response	r = 370								r = 380							
ρ	0.6		0.7		0.8		0.9		0.6		0.7		0.8		0.9	
Estimators	MSE	PRE	MSE	PRE	MSE	PRE	MSE	PRE	MSE	PRE	MSE	PRE	MSE	PRE	MSE	PRE
t_m	2.76	100	2.76	100	2.76	100	2.76	100	2.71	100	2.76	100	2.76	100	2.76	100
Scheme I																
t_{r_1}	3.69	74.79	3.41	80.97	3.14	87.89	2.88	95.64	3.90	69.51	3.33	82.88	3.06	90.15	2.81	98.32
t_{lr_1}	1.72	160.46	1.65	167.27	1.57	175.79	1.47	187.875	1.58	171.51	1.56	176.92	1.50	184.00	1.40	197.14
t_{g_1}	1.72	160.46	1.65	167.27	1.57	175.79	1.47	187.875	1.58	171.51	1.56	176.92	1.50	184.00	1.40	197.14
T_1	1.68	164.89	1.61	171.76	1.53	180.29	1.44	190.93	1.55	174.45	1.53	180.44	1.45	189.89	1.37	201.74
T_4	1.67	165.33	1.60	172.26	1.53	180.87	1.44	191.59	1.55	174.95	1.53	180.97	1.45	190.50	1.36	202.44
T_7	1.68	164.89	1.61	172.76	1.53	180.29	1.44	190.93	1.55	174.45	1.53	180.44	1.45	189.89	1.37	201.74
Scheme II																
t_{r_2}	3.94	70.14	3.62	76.34	3.31	83.37	3.02	91.34	4.09	66.33	3.47	79.67	3.17	87.00	2.89	95.32
t_{lr_2}	1.67	165.26	1.59	173.58	1.50	184.00	1.40	197.14	1.56	173.71	1.53	180.39	1.44	191.66	1.34	205.97
t_{g_2}	1.67	165.26	1.59	173.58	1.50	184.00	1.40	197.14	1.56	173.71	1.53	180.39	1.44	191.66	1.34	205.97
T_2	1.65	167.49	1.57	175.58	1.49	185.79	1.39	198.75	1.54	176.47	1.51	183.16	1.42	193.82	1.33	207.34
T_5	1.64	167.99	1.57	176.17	1.48	186.48	1.39	199.55	1.53	177.03	1.50	183.75	1.42	194.49	1.33	208.13
T_8	1.65	167.49	1.57	175.58	1.49	185.79	1.39	198.75	1.53	176.47	1.51	183.16	1.42	193.82	1.33	207.34
Scheme III																
t_{r_3}	2.12	129.92	2.09	132.29	2.00	134.63	2.02	136.92	1.96	138.58	1.94	142.61	1.91	144.36	1.89	146.07
t_{lr_3}	1.87	147.59	1.86	148.38	1.85	149.18	1.83	150.81	1.76	153.97	1.79	154.18	1.78	155.05	1.77	155.93
t_{g_3}	1.87	147.59	1.86	148.38	1.85	149.18	1.83	150.81	1.76	153.97	1.79	154.18	1.78	155.05	1.77	155.93
T_3	1.84	150.03	1.83	150.76	1.82	151.61	1.81	152.57	1.74	155.87	1.77	156.21	1.76	156.91	1.75	157.71
T_6	1.84	150.07	1.83	150.82	1.82	151.67	1.81	152.63	1.74	155.90	1.77	156.36	1.76	156.95	1.75	157.91
T_9	1.84	150.03	1.83	150.76	1.82	151.61	1.81	152.57	1.74	155.87	1.77	156.32	1.76	156.91	1.75	157.57

(ii). Compared to scheme I and scheme III, the suggested imputation approaches perform better under scheme II.
(iii). The MSE of the suggested imputation approaches reduces as ρ goes up, but the PRE increases as ρ goes up.
(iv). In addition, when the responding units increase, the MSE of the suggested imputation approaches goes down, whereas the PRE goes up.
(v). The similar inclination in the results as observed in $(i) - (iv)$ can also be observed from Table 3 to Table 9 consisting of different combinations of σ_Y, σ_X, σ_U and σ_V.

Table 4
MSE and PRE of estimators for $(\sigma_Y, \sigma_X) = (20, 30)$ and $(\sigma_U, \sigma_V) = (20, 20)$.

Response	r = 370								r = 380							
ρ	0.6		0.7		0.8		0.9		0.6		0.7		0.8		0.9	
Estimators	MSE	PRE	MSE	PRE	MSE	PRE	MSE	PRE	MSE	PRE	MSE	PRE	MSE	PRE	MSE	PRE
t_m	3.11	100	3.16	100	3.16	100	3.16	100	3.11	100	3.16	100	3.16	100	3.16	100
Scheme I																
t_{r_1}	3.39	91.97	2.92	108.39	2.66	119.12	2.40	131.67	3.29	94.38	2.83	111.83	2.56	123.30	2.31	136.79
t_{lr_1}	1.88	165.42	1.84	171.73	1.74	181.60	1.63	193.86	1.79	173.74	1.75	180.57	1.65	191.51	1.53	206.53
t_{g_1}	1.88	165.42	1.84	171.73	1.74	181.60	1.63	193.86	1.79	173.74	1.75	180.57	1.65	191.51	1.53	206.53
T_1	1.84	168.96	1.80	175.47	1.71	185.48	1.59	198.12	1.75	177.34	1.71	184.53	1.62	194.38	1.51	209.78
T_4	1.84	169.39	1.79	175.91	1.70	185.98	1.59	198.68	1.75	177.79	1.71	184.99	1.61	195.64	1.51	210.37
T_7	1.84	168.96	1.80	175.47	1.71	185.48	1.59	198.12	1.75	177.34	1.71	184.53	1.62	195.64	1.51	209.78
Scheme II																
t_{r_2}	3.56	87.55	3.02	104.69	2.72	116.16	2.43	129.85	3.41	91.30	2.89	109.24	2.60	121.22	2.33	135.51
t_{lr_2}	1.84	169.03	1.79	176.53	1.68	188.09	1.55	203.87	1.76	176.70	1.71	184.79	1.61	196.27	1.49	212.08
t_{g_2}	1.84	169.03	1.79	176.53	1.68	188.09	1.55	203.87	1.76	176.70	1.71	184.79	1.61	196.27	1.49	212.08
T_2	1.81	172.25	1.76	179.97	1.65	192.05	1.53	207.63	1.73	179.68	1.68	187.73	1.58	200.33	1.46	216.59
T_5	1.81	172.76	1.76	180.49	1.64	192.64	1.52	207.63	1.73	180.19	1.68	188.25	1.57	200.93	1.45	217.26
T_8	1.81	172.26	1.76	179.97	1.65	192.05	1.52	207.63	1.73	179.68	1.68	187.73	1.58	200.33	1.46	216.59
Scheme III																
t_{r_3}	2.29	135.87	2.26	140.11	2.22	142.35	2.19	144.58	2.14	145.28	2.13	148.41	2.11	150.03	2.08	151.63
t_{lr_3}	2.10	148.09	2.13	148.35	2.11	149.76	2.10	150.47	2.03	153.20	2.05	154.14	2.04	154.90	2.03	155.66
t_{g_3}	2.10	148.09	2.13	148.35	2.11	149.76	2.10	150.47	2.03	153.20	2.05	154.14	2.04	154.90	2.03	155.66
T_3	2.07	150.56	2.09	151.28	2.08	152.24	2.08	153.31	1.99	156.21	2.03	156.73	2.01	157.39	1.99	158.13
T_6	2.07	150.61	2.09	151.33	20.8	152.29	2.06	153.37	1.99	156.24	2.02	156.76	2.01	157.43	1.99	158.17
T_9	2.07	150.56	2.09	151.28	2.08	152.24	2.06	153.31	1.99	156.21	2.02	156.73	2.01	157.39	1.99	158.13

Table 5
MSE and PRE of estimators for $(\sigma_Y, \sigma_X) = (20, 30)$ and $(\sigma_U, \sigma_V) = (20, 25)$.

Response	r = 370								r = 380							
ρ	0.6		0.7		0.8		0.9		0.6		0.7		0.8		0.9	
Estimators	MSE	PRE	MSE	PRE	MSE	PRE	MSE	PRE	MSE	PRE	MSE	PRE	MSE	PRE	MSE	PRE
t_m	3.16	100	3.16	100	3.12	100	3.16	100	3.16	100	3.16	100	3.12	100	3.16	100
Scheme I																
t_{r_1}	3.97	79.72	3.69	85.83	3.59	86.73	3.16	100.05	3.88	81.57	3.59	87.98	3.51	88.89	3.07	102.98
t_{lr_1}	1.98	159.59	1.91	165.44	1.80	173.33	1.75	180.57	1.89	167.19	1.82	173.62	1.71	182.45	1.66	190.09
t_{g_1}	1.98	159.59	1.91	165.44	1.80	173.33	1.75	180.57	1.89	167.19	1.82	173.62	1.71	182.45	1.66	190.09
T_1	1.94	162.70	1.88	168.49	1.77	176.63	1.71	184.32	1.85	170.46	1.79	176.83	1.68	185.82	1.63	194.35
T_4	1.94	163.13	1.87	168.98	1.76	177.22	1.71	184.94	1.85	170.89	1.79	177.34	1.67	186.44	1.62	195.01
T_7	1.94	162.70	1.88	168.49	1.77	176.63	1.71	184.32	1.85	170.46	1.79	176.83	1.68	185.82	1.63	194.35
Scheme II																
t_{r_2}	4.21	75.09	3.89	81.27	3.79	82.19	3.29	95.92	4.04	78.36	3.73	84.81	3.64	85.73	3.16	100.09
t_{lr_2}	1.95	162.05	1.88	168.08	1.75	178.28	1.69	186.98	1.87	168.98	1.80	175.55	1.68	185.71	1.61	196.27
t_{g_2}	1.95	162.05	1.88	168.08	1.75	178.28	1.69	186.98	1.87	168.98	1.80	175.55	1.68	185.71	1.61	196.27
T_2	1.92	164.90	1.84	171.69	1.72	181.36	1.66	190.63	1.84	172.02	1.77	179.10	1.65	189.19	1.59	198.86
T_5	1.91	165.39	1.84	172.26	1.71	182.06	1.65	191.38	1.83	172.50	1.76	179.67	1.64	189.89	1.58	199.60
T_8	1.92	164.90	1.84	171.69	1.72	181.36	1.66	190.63	1.84	172.02	1.77	179.10	1.65	189.19	1.59	198.86
Scheme III																
t_{r_3}	2.39	131.83	2.36	133.96	2.33	134.26	2.29	138.10	2.22	142.26	2.19	143.87	2.17	144.08	2.15	146.94
t_{lr_3}	2.15	146.97	2.14	147.66	2.10	148.57	2.11	149.76	2.07	152.65	2.06	153.39	2.03	153.69	2.04	154.90
t_{g_3}	2.15	146.97	2.14	147.66	2.10	148.57	2.11	149.76	2.07	152.65	2.06	153.39	2.03	153.69	2.04	154.90
T_3	2.11	149.92	2.10	150.56	2.06	151.36	2.08	152.12	2.03	155.78	2.02	156.23	1.99	156.77	2.01	157.31
T_6	2.11	149.97	2.10	150.61	2.06	151.42	2.08	152.19	2.03	155.81	2.02	156.26	1.99	156.81	2.01	157.35
T_9	2.11	149.92	2.10	150.56	2.06	151.36	2.08	152.12	2.03	155.78	2.02	156.23	1.99	156.77	2.01	157.31

7. Conclusion

In the present paper, we have developed few efficient methods of imputation to sort out the issue of missing values provided that the data are contaminated with ME as well. The 1^{st} order approximated formula of MSE of the corresponding resultant estimators are obtained. A comparative study is performed under which the developed imputations dominate the existing imputations. A numerical study is performed utilizing real populations. The numerical findings exhibited in Table 1 show that the developed imputations repress the conventional imputation methods with least MSE and hightest PRE. Moreover, a simulation analysis is executed on a hypothetically rendered normal population using different values of σ_Y, σ_X, σ_U, σ_V, ρ and responding units r. We consider different values of σ_Y, σ_X, σ_U, σ_V, ρ and r to notice the tendency of the proposed imputations. Tables 2–9 display the simulation findings that

Table 6
MSE and PRE of estimators for $(\sigma_Y, \sigma_X)=(20, 35)$ and $(\sigma_U, \sigma_V)=(15, 20)$.

Response	r = 370								r = 380							
ρ	0.6		0.7		0.8		0.9		0.6		0.7		0.8		0.9	
Estimators	MSE	PRE	MSE	PRE	MSE	PRE	MSE	PRE	MSE	PRE	MSE	PRE	MSE	PRE	MSE	PRE
t_m	3.52	100	3.58	100	3.53	100	3.57	100	3.52	100	3.57	100	3.53	100	3.57	100
Scheme I																
t_{r_1}	3.44	102.48	2.95	121.38	2.72	129.64	2.32	154.12	3.34	105.46	2.84	125.66	2.62	134.52	2.21	161.14
t_{lr_1}	3.04	172.79	1.98	180.69	1.79	196.25	1.69	211.35	1.94	181.69	1.88	190.46	1.69	207.82	1.59	224.84
t_{g_1}	2.04	172.79	1.98	180.69	1.79	196.25	1.69	211.35	1.94	181.69	1.88	190.46	1.69	207.82	1.59	224.84
T_1	2.02	174.16	1.96	182.11	1.79	197.62	1.68	212.76	1.92	183.06	1.86	191.87	1.69	209.19	1.58	226.25
T_4	2.02	174.64	1.96	182.59	1.78	198.22	1.67	213.36	1.92	183.57	1.86	192.38	1.68	209.83	1.57	226.88
T_7	2.02	174.64	1.96	182.11	1.79	197.62	1.68	212.76	1.92	183.06	1.86	191.87	1.69	209.19	1.58	226.25
Scheme II																
t_{r_2}	3.58	98.46	3.01	118.57	2.77	127.63	2.30	155.17	3.43	102.65	2.89	123.70	2.65	133.09	2.20	161.88
t_{lr_2}	1.99	177.02	1.92	186.49	1.72	205.60	1.59	224.69	1.91	184.71	1.84	194.60	1.64	214.53	1.52	234.45
t_{g_2}	1.99	177.02	1.92	186.49	1.72	205.60	1.59	224.69	1.91	184.71	1.84	194.60	1.64	214.53	1.52	234.45
T_2	1.98	178.02	1.90	187.92	1.71	206.97	1.58	226.10	1.89	186.07	1.83	196.02	1.63	215.89	1.51	235.86
T_5	1.97	178.95	1.89	188.49	1.69	207.69	1.57	226.84	1.89	186.64	1.82	196.59	1.63	216.62	1.51	236.59
T_8	1.98	178.39	1.90	187.92	1.71	206.97	1.58	226.10	1.89	186.07	1.82	196.02	1.63	215.89	1.51	235.86
Scheme III																
t_{r_3}	2.54	138.71	2.50	142.78	2.45	144.24	2.42	147.77	2.39	147.37	2.38	150.33	2.33	151.38	2.32	153.89
t_{lr_3}	2.35	149.87	2.37	150.64	2.32	151.99	2.33	153.14	2.27	155.36	2.29	155.89	2.25	156.84	2.26	157.63
t_{g_3}	2.35	149.87	2.37	150.64	2.32	151.99	2.33	153.14	2.27	155.36	2.29	155.89	2.25	156.84	2.26	157.63
T_3	2.33	151.24	2.35	152.07	2.30	153.38	2.31	154.56	2.25	156.73	2.27	157.32	2.23	158.21	2.24	159.04
T_6	2.33	151.29	2.35	152.12	2.30	153.43	2.31	154.61	2.25	156.76	2.27	157.35	2.23	158.25	2.24	159.08
T_9	2.33	151.24	2.35	152.07	2.30	153.38	2.31	154.56	2.25	156.73	2.27	157.32	2.23	158.21	2.24	159.04

Table 7
MSE and PRE of estimators for $(\sigma_Y, \sigma_X)=(20, 35)$ and $(\sigma_U, \sigma_V)=(15, 25)$.

Response	r = 370								r = 380							
ρ	0.6		0.7		0.8		0.9		0.6		0.7		0.8		0.9	
Estimators	MSE	PRE	MSE	PRE	MSE	PRE	MSE	PRE	MSE	PRE	MSE	PRE	MSE	PRE	MSE	PRE
t_m	3.52	100	3.58	100	3.53	100	3.57	100	3.52	100	3.52	100	3.53	100	3.57	100
Scheme I																
t_{r_1}	4.34	81.27	3.74	95.65	3.53	98.86	3.09	115.39	4.24	83.11	3.85	91.56	3.47	101.64	2.99	119.27
t_{lr_1}	2.15	163.72	2.11	169.66	1.98	178.28	1.88	189.89	2.05	171.70	1.96	179.59	1.86	189.78	1.78	200.56
t_{g_1}	2.15	163.72	2.11	169.66	1.98	178.28	1.88	189.89	2.05	171.70	1.96	179.59	1.86	189.78	1.78	200.56
T_1	2.10	167.67	2.06	173.23	1.92	184.14	1.84	194.01	2.00	175.90	1.92	184.03	1.81	194.14	1.74	205.15
T_4	2.09	168.19	2.06	173.77	1.91	184.81	1.83	194.70	1.99	176.46	1.91	184.66	1.81	194.84	1.73	205.88
T_7	2.10	167.67	2.06	173.23	1.92	184.14	1.84	194.01	2.00	175.90	1.92	184.03	1.81	194.14	1.74	205.15
Scheme II																
t_{r_2}	4.59	76.65	3.91	91.35	3.73	94.68	3.18	112.14	4.41	79.90	3.98	88.44	3.57	98.73	3.05	116.99
t_{lr_2}	2.11	166.82	2.06	173.78	1.90	185.78	1.81	197.23	2.02	174.25	1.92	183.33	1.81	195.02	1.73	206.35
t_{g_2}	2.11	166.82	2.06	173.78	1.90	185.78	1.81	197.23	2.02	174.25	1.92	183.33	1.81	195.02	1.73	206.35
T_2	2.06	170.71	2.02	177.27	1.85	190.39	1.76	202.48	1.98	178.06	1.88	187.16	1.78	198.60	1.69	211.21
T_5	2.06	171.32	2.01	177.91	1.85	191.19	1.76	203.32	1.97	178.67	1.88	187.86	1.77	199.39	1.68	212.04
T_8	2.06	170.71	2.02	177.27	1.85	190.39	1.76	202.48	1.98	178.06	1.88	187.16	1.78	198.60	1.69	211.21
Scheme III																
t_{r_3}	2.66	132.39	2.61	136.93	2.56	137.79	2.52	141.61	2.47	142.67	2.44	144.68	2.41	146.69	2.39	149.49
t_{lr_3}	2.39	147.28	2.41	148.54	2.37	148.94	2.38	150.00	2.30	153.04	2.29	153.71	2.29	154.14	2.31	154.54
t_{g_3}	2.39	147.28	2.41	148.54	2.37	148.94	2.38	150.00	2.30	153.04	2.29	153.71	2.29	154.14	2.31	154.54
T_3	2.34	150.56	2.36	151.19	2.32	152.21	2.33	153.11	2.25	156.25	2.25	156.79	2.24	157.40	2.26	158.04
T_6	2.34	150.62	2.36	151.25	2.32	152.28	2.33	153.18	2.25	156.29	2.25	156.83	2.24	156.45	2.26	158.09
T_9	2.34	150.56	2.36	151.19	2.32	152.21	2.33	153.11	2.25	156.25	2.25	156.79	2.24	157.40	2.26	158.04

indicate the supremacy of the developed imputations against the mean imputation, classical ratio and regression imputations and the imputations suggested by [30] with least MSEs and greatest PREs for various correlation coefficients and response rates values. Thus, the developed imputations may be considered by the survey practitioners for estimating $\bar{Y}$ given the data are missing as well as contaminated with ME.

In addition, these imputation methods and the resulting estimators may also be examined by utilizing ranked set sampling and its different variations. For more details, see [32], [33], [34], [35] and [36].

Table 8
MSE and PRE of estimators for $(\sigma_Y, \sigma_X) = (20, 35)$ and $(\sigma_U, \sigma_V) = (20, 20)$.

Response	r = 370								r = 380							
ρ	0.6		0.7		0.8		0.9		0.6		0.7		0.8		0.9	
Estimators	MSE	PRE	MSE	PRE	MSE	PRE	MSE	PRE	MSE	PRE	MSE	PRE	MSE	PRE	MSE	PRE
t_m	3.92	100	3.97	100	3.93	100	3.97	100	3.92	100	3.97	100	3.93	100	3.97	100
Scheme I																
t_{r_1}	3.71	105.73	3.22	123.51	2.99	131.20	2.59	153.36	3.60	108.91	3.10	127.97	2.88	136.20	2.47	160.31
t_{lr_1}	2.31	169.73	2.25	176.59	2.07	189.76	1.96	202.40	2.19	178.33	2.14	185.92	1.96	200.56	1.85	214.75
t_{g_1}	2.31	169.73	2.25	176.59	2.07	189.76	1.96	202.40	2.19	178.33	2.14	185.92	1.96	200.56	1.85	214.75
T_1	2.29	171.26	2.23	178.18	2.05	191.29	1.95	203.98	2.18	179.85	2.12	187.49	1.94	202.09	1.85	216.31
T_4	2.28	171.73	2.22	178.65	2.05	191.86	1.94	204.54	2.17	180.34	2.11	187.99	1.94	202.69	1.83	216.91
T_7	2.29	171.26	2.23	178.18	2.05	191.29	1.95	203.98	2.18	179.85	2.12	187.49	1.94	202.09	1.83	216.31
Scheme II																
t_{r_2}	3.85	101.87	3.29	120.91	3.04	129.34	2.57	154.29	3.69	106.22	3.15	126.14	2.91	134.89	2.46	160.96
t_{lr_2}	2.26	173.39	2.19	181.57	1.99	197.57	1.86	213.32	2.17	180.93	2.09	189.46	1.90	206.15	1.78	222.58
t_{g_2}	2.26	173.39	2.19	181.57	1.99	197.57	1.86	231.32	2.17	180.93	2.09	189.46	1.90	206.15	1.78	222.58
T_2	2.24	174.92	2.17	183.15	1.97	199.09	1.85	214.89	2.15	182.45	2.08	191.04	1.89	207.67	1.77	224.15
T_5	2.24	175.47	2.16	183.70	1.97	199.78	1.84	215.57	2.14	182.99	2.07	191.59	1.88	208.36	1.76	224.83
T_8	2.24	174.92	2.17	183.15	1.97	199.09	1.85	214.89	2.15	182.44	2.08	191.04	1.89	207.67	1.77	224.15
Scheme III																
t_{r_3}	2.81	139.49	2.78	143.17	2.72	144.50	2.69	147.68	2.65	147.95	2.64	150.62	2.59	151.56	2.58	153.82
t_{lr_3}	2.62	149.55	2.65	150.25	2.59	151.46	2.60	152.48	2.53	155.14	2.55	155.63	2.51	156.47	2.52	157.18
t_{g_3}	2.62	149.55	2.65	150.25	2.59	151.46	2.60	152.48	2.53	155.14	2.55	155.63	2.51	156.47	2.52	157.18
T_3	2.59	151.13	2.62	151.83	2.57	152.99	2.58	154.06	2.50	156.66	2.53	157.20	2.49	157.99	2.49	158.74
T_6	2.59	151.13	2.62	151.88	2.57	153.04	2.58	154.11	2.50	156.69	2.53	157.24	2.49	158.03	2.49	158.78
T_9	2.59	151.08	2.62	151.83	2.57	152.99	2.58	154.06	2.50	156.66	2.53	157.20	2.49	157.99	2.49	158.74

Table 9
MSE and PRE of estimators for $(\sigma_Y, \sigma_X) = (20, 35)$ and $(\sigma_U, \sigma_V) = (20, 25)$.

Response	r = 370								r = 380							
ρ	0.6		0.7		0.8		0.9		0.6		0.7		0.8		0.9	
Estimators	MSE	PRE	MSE	PRE	MSE	PRE	MSE	PRE	MSE	PRE	MSE	PRE	MSE	PRE	MSE	PRE
t_m	3.97	100	3.93	100	3.93	100	3.97	100	3.97	100	3.92	100	3.93	100	3.97	100
Scheme I																
t_{r_1}	4.35	91.29	4.22	93.02	3.84	102.25	3.37	117.93	4.24	93.70	4.11	95.47	3.73	105.24	3.52	121.98
t_{lr_1}	2.45	162.50	2.30	170.49	2.20	178.33	2.13	186.74	2.33	170.37	2.19	179.16	2.09	187.84	2.01	197.20
t_{g_1}	2.45	162.50	2.30	170.49	2.20	178.33	2.13	186.74	2.33	170.37	2.19	179.16	2.09	187.84	2.01	197.20
T_1	2.42	164.09	2.28	172.02	2.18	179.86	2.11	188.32	2.31	171.94	2.17	180.68	2.07	189.37	1.99	198.77
T_4	2.42	164.54	2.27	172.59	2.18	180.50	2.10	188.98	2.30	172.42	2.17	181.29	2.07	190.04	1.99	199.46
T_7	2.42	164.09	2.28	172.02	2.18	179.86	2.11	188.32	2.31	171.94	2.17	180.68	2.07	189.37	1.99	198.77
Scheme II																
t_{r_2}	4.58	86.86	4.43	88.63	4.00	98.22	2.46	114.87	4.38	90.61	4.24	92.41	3.83	102.43	3.31	119.84
t_{lr_2}	2.41	164.88	2.25	174.29	2.14	183.66	2.05	193.87	2.31	172.05	2.16	181.86	2.05	191.64	1.96	202.29
t_{g_2}	2.41	164.88	2.25	174.29	2.14	183.66	2.05	193.87	2.31	172.05	2.16	181.86	2.05	191.64	1.96	202.29
T_2	2.39	166.47	2.23	175.81	2.12	185.19	2.03	195.44	2.29	173.63	2.14	183.38	2.03	193.16	1.95	203.86
T_5	2.38	166.99	2.22	176.49	2.11	185.95	2.02	195.23	2.28	174.16	2.13	184.06	2.03	193.92	1.94	204.65
T_8	2.89	166.47	2.23	175.81	2.12	185.19	2.03	195.44	2.29	173.63	2.14	183.38	2.03	193.16	1.95	203.86
Scheme III																
t_{r_3}	2.93	135.67	2.88	136.18	2.83	138.65	2.79	142.12	2.74	145.14	2.69	145.50	2.67	147.33	2.65	149.86
t_{lr_3}	2.67	148.76	2.62	149.63	2.61	150.42	2.63	151.19	2.57	154.59	2.53	155.19	2.52	155.74	2.54	156.28
t_{g_3}	2.67	148.76	2.62	149.63	2.61	150.42	2.63	151.19	2.57	154.59	2.53	155.19	2.52	155.74	2.54	156.28
T_3	2.64	150.35	2.59	151.16	2.59	151.95	2.59	152.77	2.54	156.17	2.50	156.72	2.49	157.27	2.51	157.85
T_6	2.64	150.40	2.59	151.22	2.59	152.01	2.59	152.83	2.54	156.21	2.50	156.76	2.49	157.31	2.51	157.89
T_9	2.64	150.35	2.59	151.16	2.59	151.95	2.59	152.77	2.54	156.17	2.50	156.72	2.49	157.27	2.51	157.85

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CRediT authorship contribution statement

Anoop Kumar: Writing – original draft, Validation, Software, Methodology, Formal analysis, Data curation, Conceptualization. **Shashi Bhushan:** Writing – review & editing, Validation, Supervision, Methodology, Investigation, Conceptualization. **Shivam**

Shukla: Software, Formal analysis, Data curation. **M.E. Bakr:** Visualization, Project administration, Funding acquisition. **Arwa M. Alshangiti:** Investigation, Funding acquisition, Data curation. **Oluwafemi Samson Balogun:** Software, Resources, Project administration.

Declaration of competing interest

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Data availability

Data included in article/supplementary material/referenced in article.

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Appendix A

This part presents the MSE and optimum scalar values of the resulting estimators consisting of the various imputation techniques covered in Section 3.

$$MSE(t_m) = f_r \tau_y S_Y^2$$
$$MSE(t_{r_1}) = \mu_Y^2 (f_r \tau_y C_Y^2 + f_n \tau_x C_X^2 - 2 f_n \rho C_Y C_X)$$
$$MSE(t_{r_2}) = \mu_Y^2 (f_r \tau_y C_Y^2 + f_r \tau_x C_X^2 - 2 f_r \rho C_Y C_X)$$
$$MSE(t_{r_3}) = \mu_Y^2 (f_r \tau_y C_Y^2 + f_{rn} \tau_x C_X^2 - 2 f_{rn} \rho C_Y C_X)$$
$$minMSE(t_{lr_1}) = \mu_Y^2 C_Y^2 \left(f_r \tau_y - f_n \frac{\rho^2}{\tau_x} \right)$$
$$minMSE(t_{lr_2}) = f_r \mu_Y^2 C_Y^2 \left(\tau_y - \frac{\rho^2}{\tau_x} \right)$$
$$minMSE(t_{lr_3}) = \mu_Y^2 C_Y^2 \left(f_r \tau_y - f_{rn} \frac{\rho^2}{\tau_x} \right)$$

The optimum value of scalars $\beta_i,\ i = 1, 2, 3$ are respectively given below.

$$\beta_i = \rho \frac{C_Y}{\tau_x C_X}$$

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