Chapter 3

E-Bayesian Inference for Poisson Inverse Exponential Distribution Under Different Loss Functions *

3.1 Introduction

In the previous chapter, we have discussed the procedure for obtaining the classical and Bayesian estimation under PT-II CBRs. This chapter, we obtain the E- Bayes estimator of the considered Poisson family, we know that the distribution as a Poisson inverse exponential distribution (PIED). Here, experimental units generally deal with truncated data in studies pertaining to medical and survival analysis. However, data might be a set of some samples or censored data from an experiment. The literature has witnessed the different type of censoring techniques to effectively reduce the time and cost of the life testing experimentation. The Type-I and Type-II censoring are the very useful real life problems. Type-I (pertains to time constraint) and Type-II (pertains to cost constraint) censoring; time and number of experimental units are pre-specified

*Part of this chapter has been published in reputed peer-reviewed journals with indexing SCIE, SCOPUS, PubMed see Pathak et al. (2020a).

or fixed, which has been controlled by experimenter. However, it may not be always true for data related with medical studies. Because some of the surviving experimental units are removed randomly at each stage of failure from the experiment due to some unforeseen reasons, which are beyond the control of the experimenter. For example, let us suppose that n, number of multiple myeloma patients have a malignant disease characterized by the accumulation of abnormal plasma cell, a type of white blood cell, in the bone marrow, and are put under medication in hospital. It is decided to observe the survival lifetime of m patients out of n. During medication it may happens that after death (first death, second death or so on) some patients may leave the hospital due to various reasons like loss in faith with hospital, treatment etc. The process of taking observations continues till survival times of m patients are recorded. It may also be noted that the number of patients dropping out from the test at each stage is random and can not be predetermined. The above said censoring and their mathematical formulation, expression have been discussed in Chapter 1, Subsection 1.11.2.

In last few years, the estimation of parameters of different lifetime distribution based on progressive censored samples have been discussed by several authors such as (Childs and Balakrishnan (2000), Kundu (2008), Kim and Han (2009), Gholizadeh et al. (2011), Kim et al. (2011), Huang and Wu (2012)).

Some early works based on the estimation of parameters of different lifetime distribution under PT-II CBRs has been done by (Tse and Yuen (2000), Wu and Chang (2002), Singh et al. (2013b), Singh et al. (2014), Kumar et al. (2015), Kumar et al. (2018), Kumar et al. (2019a)).

Firstly, han has discussed the E-Bayesian inference, which is an alternative to Bayesian inference. This method has been used as prior for the unknown hyper parameters. The hierarchical prior distribution may be used as prior for the unknown hyper parameters, and has required to set the at least two stages of prior setting (see Lindley and Smith (1972)). But in practice, under censoring mechanism through this prior, the Bayesian estimates of the unknown parameters that have been obtained is quite complicated for the purpose of data analysis as well as computations. For this contrary, we are approaching to follow E-Bayesian estimation method for PIED parameters under PT-II CBRs.

The detailed discussions about the E-Bayesian method see, (Han (2007), Han (2011a), Han (2011b)). The E-Bayesian estimation for the parameters of different lifetime distributions (see Gupta (2017), Yousefzadeh (2017), Han (2017a), Han (2017b), Han (2019b) etc.). Furthermore, a few number of authors dealt with the E-Bayesian estimation of parameters for lifetime distribution with type-II censoring (see Jaheen and Okasha (2011), Okasha (2014), Reyad and Ahmed (2016) etc.).

Also, El-Sagheer (2017) considered the Rayleigh distribution for E-Bayesian estimation under progressive type-II censoring. The hierarchical and E-Bayesian estimations for the proportional reversed hazard rate model based on record values have been discussed by Kızılaslan (2017). Moreover, some more relevant literature related to the study of E-Bayesian estimation for E-posterior and E-MSE has been done (see Han (2018), Han (2019a), Han (2019b), Han (2019c), Han (2020)).

Recently, PIED as a parametric compounding based upon Poisson lifetime distribution which has been introduced by Kumar et al. (2018). The various parametric compounding based on Poisson lifetime distributions ((Barreto-Souza and Cribari-Neto (2009), Louzada-Neto et al. (2011), Lu and Shi (2012), Kumar et al. (2018)) have been used for parameter estimation, but no one attempted to work on E-Bayesian inference for the parameters under PT-II CBRs. This is the beauty of this chapter.

Finally, in this chapter we obtain the E-Bayesian and Bayesian estimators of parameters of PIED under SELF, GELF and LINEX for PT-II CBRs. The E-Bayesian estimators are compared with Bayesian estimators and obtained under different loss function in terms of their risks.

3.2 Mathematical Formulation

We begin by summarizing PIED model and likelihood under PT-II CBRs along with Bayesian and E-Bayesian approach that will be used through out the chapter.

3.2.1 The Model

The two-parameter PIED is one of the latest compounding of two most useful probability distributions termed as Poisson inverse exponential distribution i.e., zero truncated Poisson and inverse exponential, and their PDF is

$$f(x;\theta,\lambda) = \frac{\theta\lambda e^{-\theta-\frac{\lambda}{x}+\theta e^{-\frac{\lambda}{x}}}}{\left(1-e^{-\theta}\right)x^2}; \qquad x>0, \lambda>0, \theta>0, \tag{3.1}$$

where, the parameter θ and λ are represented as shape and scale, respectively. The corresponding CDF of PIED (λ,θ) is given by

$$F(x;\theta,\lambda) = \frac{e^{-\theta}\left(e^{\theta e^{-\frac{\lambda}{x}}}-1\right)}{1-e^{-\theta}}; \qquad x>0, \lambda>0, \theta>0. \tag{3.2}$$

It is a lifetime distribution with initially increasing then decreasing failure distribution. The HF can be defined as

$$h(x;\theta,\lambda) = \frac{f(x;\theta,\lambda)}{1-F(x;\theta,\lambda)} = \frac{\theta\lambda e^{-\theta-\frac{\lambda}{x}+\theta e^{-\frac{\lambda}{x}}}}{x^2\left(1-e^{-\theta+\theta e^{-\frac{\lambda}{x}}}\right)}; \qquad x>0, \lambda>0, \theta>0. \tag{3.3}$$

It is very much plausible statistical distribution and alternative to common mixture of lifetime distributions when it is heavy-tailed with monotone failure data. As per applied mathematical formulation of PT-II CBR mechanism are discussed in Chapter 1, Section 1.11.2. The conditional likelihood function can be written as (see Cohen (1963) and Kamps and Cramer (2001))

$$\begin{aligned} L(\lambda,\theta;x|R=r) &= f_{(X_1,\cdots,X_m)}(x_1,\cdots,x_m) \\ &= c\prod_{i=1}^{m} f(x_i)\left[1-F(x_i)\right]^{r_i}; \quad -\infty < x_1 < \cdots < x_m < \infty, \end{aligned} \tag{3.4}$$

where $n = m + \sum_{i=1}^{m} r_i, n, m \in N, r_i \in N_0, 1 \le i \le m, r_i \sim B(n-m-\sum_{l=0}^{i-1} r_l, p)$ for $i = 1, 2, 3, \cdots, m-1$ and $r_0 = 0$ and $c = \prod_{i=1}^{m} \gamma_i$ with $\gamma_i = \sum_{j=i}^{m}(r_j+1)$ and for $\gamma_1 = n$. Substituting $f(x_i)$ and $F(x_i)$ from Equations (3.1) and (3.2) into Equation (3.4), it reduces to

$$L(\lambda,\theta;x|R=r) = c\prod_{i=1}^{m} \frac{\theta\lambda e^{-\theta-\frac{\lambda}{x_i}+\theta e^{-\frac{\lambda}{x_i}}}}{(1-e^{-\theta})x_i^2}\left\{1-\frac{e^{-\theta}\left(e^{\theta e^{-\frac{\lambda}{x_i}}}-1\right)}{(1-e^{-\theta})}\right\}^{r_i}. \tag{3.5}$$

The number of the experimental unit R_i removed at i^{th} failure X_i; $i = 1, 2, \cdots (m-1)$, follows a Binomial distribution with parameters $\left(n-m-\sum_{l=1}^{i-1} r_i, p\right)$. Therefore,

$$P(R_1 = r_1; p) = \binom{n-m}{r_1} p^{r_1}(1-p)^{n-m-r_1}, \tag{3.6}$$

and for $i = 2, 3, \cdots, m-1$,

$$\begin{aligned} P(R_i; p) &= P(R_i = r_i | R_{i-1} = r_{i-1}, \cdots R_1 = r_1) \\ &= \binom{n-m-\sum_{l=0}^{i-1} r_l}{r_i} p^{r_i}(1-p)^{n-m-\sum_{l=0}^{i-1} r_l}. \end{aligned} \tag{3.7}$$

We also assume that $R_i s$ are independent of X_i for all i. Thus, the joint likelihood function $X_i, i = 1, 2, 3, \cdots, m$ and $R_i,\ i = 1, 2, 3, \cdots, m$ can take the following form

$$L(\theta,\lambda,p;x) = L(\theta,\lambda;x|R=r)\,P(R=r;p), \tag{3.8}$$

where

$$P(R=r;p)=P(R_1=r_1)P(R_2=r_2|R_1=r_1)P(R_3=r_3|R_2=r_2,R_1=r_1) \\ \cdots P(R_{m-1}=r_{m-1}|R_{m-2}=r_{m-2},\cdots R_1=r_1). \tag{3.9}$$

Substituting Equations (3.6) and (3.7) into Equation (3.9), we get

$$P(R=r;p)=\frac{(n-m)!p^{\sum_{i=1}^{m-1}r_i}(1-p)^{(m-1)(n-m)-\sum_{i=1}^{m-1}(m-i)r_i}}{\left(n-m-\sum_{l=1}^{i-1}r_i\right)!\prod_{i=1}^{m-1}r_i!}. \tag{3.10}$$

Now using Equations (3.5), (3.8) and (3.10) the complete likelihood function can be written as

$$L(\lambda,\theta,p;x)=\eta L_1(\lambda,\theta)L_2(p), \tag{3.11}$$

where $\eta=\frac{c(n-m)!}{(n-m-\sum_{l=1}^{i-1}r_i)!\prod_{i=1}^{m-1}r_i!}$,

$$L_1(\lambda,\theta)=\prod_{i=1}^{m}\frac{\theta\lambda e^{-\theta-\frac{\lambda}{x_i}+\theta e^{-\frac{\lambda}{x_i}}}}{(1-e^{-\theta})x_i{}^2}\left\{1-\frac{e^{-\theta}\left(e^{\theta e^{-\frac{\lambda}{x_i}}}-1\right)}{(1-e^{-\theta})}\right\}^{r_i}, \tag{3.12}$$

and

$$L_2(p)=p^{\sum_{i=1}^{m-1}r_i}(1-p)^{(m-1)(n-m)-\sum_{i=1}^{m-1}(m-i)r_i}. \tag{3.13}$$

3.3 Maximum Likelihood Estimation under PT-II CBRs

3.3.1 Point Estimation

As we observed that $L_1(\lambda,\theta)$ has independent of $L_2(p)$. Therefore, the ML estimates of λ and θ can derived by maximizing Equation (3.12) directly. The log-L function of the above

Equation (3.12) becomes

$$\begin{aligned}\ln L_1(\lambda,\theta) = -m\theta + m\ln(\theta) + m\ln(\lambda) - \lambda\sum_{i=1}^{m}\frac{1}{x_i} + \theta\sum_{i=1}^{m} e^{-\frac{\lambda}{x_i}} \\ -\left(m+\sum_{i=1}^{m} r_i\right)\ln\left(1-e^{-\theta}\right) - \sum_{i=1}^{m} 2\ln(x_i) + \sum_{i=1}^{m} r_i \ln\left(1-e^{-\theta\left(1-e^{-\frac{\lambda}{x_i}}\right)}\right).\end{aligned} \tag{3.14}$$

The ML estimates of (λ,θ) can be directly obtained by maximizing the log-L function Equation (3.14), or alternatively, by finding the solution for the following two nonlinear Equations,

$$\frac{m}{\lambda} - \sum_{i=1}^{m}\frac{1}{x_i} - \theta\sum_{i=1}^{m}\frac{1}{x_i}e^{-\frac{\lambda}{x_i}} - \sum_{i=1}^{m}\frac{r_i e^{-\frac{\lambda}{x_i}}}{x_i\left(e^{\theta\left(1-e^{-\frac{\lambda}{x_i}}\right)}-1\right)} = 0, \tag{3.15}$$

and

$$\frac{m}{\theta} - m + \sum_{i=1}^{m} e^{-\frac{\lambda}{x_i}} - \left(m+\sum_{i=1}^{m} r_i\right)\frac{e^{-\theta}}{1-e^{-\theta}} - \sum_{i=1}^{m}\frac{r_i(1-e^{-\frac{\lambda}{x_i}})}{\left(e^{\theta\left(1-e^{-\frac{\lambda}{x_i}}\right)}-1\right)} = 0. \tag{3.16}$$

The above Equations (3.15) and (3.16) can not be solved simultaneously to provide any explicit solution for $\Psi = (\lambda,\theta)$. Therefore, these normal equations are to be solved numerically using some adequate iteration such as the NR method or an algorithm such as *nlm* of software R (Ihaka and Gentleman (1996)).

3.3.2 Confidence Intervals

Now, we discussed CIs of the parameters λ and θ under PT-II CBRs. Therefore we have,

$$\begin{bmatrix} Var(\hat{\lambda}_M) & Cov(\hat{\lambda}_M,\hat{\theta}_M) \\ Cov(\hat{\lambda}_M,\hat{\theta}_M) & Var(\hat{\theta}_M) \end{bmatrix} = \begin{bmatrix} -\frac{\partial^2 \ln L_1(\lambda,\theta)}{\partial\lambda^2} & -\frac{\partial^2 \ln L_1(\lambda,\theta)}{\partial\lambda\partial\theta} \\ -\frac{\partial^2 \ln L_1(\lambda,\theta)}{\partial\theta\partial\lambda} & -\frac{\partial^2 \ln L_1(\lambda,\theta)}{\partial\theta^2} \end{bmatrix}^{-1}_{\lambda=\hat{\lambda}_M,\theta=\hat{\theta}_M}, \tag{3.17}$$

where,

$$\frac{\partial^2 \ln L_1(\lambda,\theta)}{\partial\lambda^2} = -\frac{m}{\lambda^2} + \theta\sum_{i=1}^{m}\left(\frac{1}{x_i}\right)^2 e^{-\frac{\lambda}{x_i}} - \sum_{i=1}^{m} T_i,$$

and,

$$T_i = \frac{r_i e^{-\frac{\lambda}{x_i}}\left(1 - e^{\theta\left(1-e^{-\frac{\lambda}{x_i}}\right)} - e^{\theta\left(1-e^{-\frac{\lambda}{x_i}}\right)} e^{-\frac{\lambda}{x_i}}\right)}{x_i^2\left(e^{\theta\left(1-e^{-\frac{\lambda}{x_i}}\right)} - 1\right)^2},$$

$$\frac{\partial^2 \ln L_1(\lambda,\theta)}{\partial\lambda\partial\theta} = -\sum_{i=1}^{m}\frac{e^{-\frac{\lambda}{x_i}}}{x_i} + \sum_{i=1}^{m}\frac{r_i e^{-\frac{\lambda}{x_i}} e^{\theta\left(1-e^{-\frac{\lambda}{x_i}}\right)}\left(1-e^{-\frac{\lambda}{x_i}}\right)}{x_i\left(e^{\theta\left(1-e^{-\frac{\lambda}{x_i}}\right)} - 1\right)^2},$$

$$\frac{\partial^2 \ln L_1(\lambda,\theta)}{\partial\theta\partial\lambda} = -\sum_{i=1}^{m}\frac{e^{-\frac{\lambda}{x_i}}}{x_i} + \sum_{i=1}^{m}\frac{r_i e^{-\frac{\lambda}{x_i}}\left(e^{\theta\left(1-e^{-\frac{\lambda}{x_i}}\right)}\left(\theta(1-e^{-\frac{\lambda}{x_i}})-1\right)+1\right)}{x_i\left(e^{\theta\left(1-e^{-\frac{\lambda}{x_i}}\right)} - 1\right)^2},$$

$$\frac{\partial^2 \ln L_1(\lambda,\theta)}{\partial\theta^2} = -\frac{m}{\theta^2} + \left(m + \sum_{i=1}^{m} r_i\right)\frac{e^{-\theta}}{(1-e^{-\theta})^2} + \sum_{i=1}^{m}\frac{r_i e^{\theta\left(1-e^{-\frac{\lambda}{x_i}}\right)}\left(1-e^{-\frac{\lambda}{x_i}}\right)^2}{\left(e^{\theta\left(1-e^{-\frac{\lambda}{x_i}}\right)} - 1\right)^2}.$$

The $\hat{\lambda}_M$ and $\hat{\theta}_M$ are denoted as ML estimates of λ and θ. For the asymptomatic variance-covariance of λ and θ are computed by invert og the Fisher's information matrix,

$$I = E\begin{bmatrix} -\frac{\partial^2 \ln L_1(\lambda,\theta)}{\partial\lambda^2} & -\frac{\partial^2 \ln L_1(\lambda,\theta)}{\partial\lambda\partial\theta} \\ -\frac{\partial^2 \ln L_1(\lambda,\theta)}{\partial\theta\partial\lambda} & -\frac{\partial^2 \ln L_1(\lambda,\theta)}{\partial\theta^2} \end{bmatrix}.$$

Thus, an approximate $100(1-\alpha)\%$ CIs for the parameters λ and θ are given by

$$\left(\hat{\lambda}_M - z_{\alpha/2}\sqrt{Var(\hat{\lambda}_M)}, \qquad \hat{\lambda}_M + z_{\alpha/2}\sqrt{Var(\hat{\lambda}_M)}\right)$$

and

$$\left(\hat{\theta}_M - z_{\alpha/2}\sqrt{Var(\hat{\theta}_M)}, \qquad \hat{\theta}_M + z_{\alpha/2}\sqrt{Var(\hat{\theta}_M)}\right),$$

respectively, where $z_{\alpha/2}$ is the percentile $(\alpha/2)^{th}$ of the standard normal distribution, also $Var(\hat{\lambda}_M)$ and $Var(\hat{\theta}_M)$ represent asymptotic variances of ML estimates.

3.4 Bayesian and E-Bayesian Estimation

We discuss the process of obtaining the Bayesian and E-Bayesian estimator of PIED parameters λ and θ based under PT-II CBRs. Bayesian and E-Bayesian estimators of the respective parameters are shown below.

3.4.1 Bayesian Estimation

Based on PT-II CBRs, observations from the PIED, the likelihood function given by Equation (3.12), prior distributions for the parameters in the distribution is $g_1(\lambda|a,b)$ of λ is given by Equation (3.18) and $g_2(\theta|\alpha,\beta)$ of θ is given by Equation (3.19) respectively,

$$g_1(\lambda|a,b) = \frac{b^a}{\Gamma(a)} e^{-b\lambda} \lambda^{a-1}; \qquad \lambda > 0, a > 0, b > 0, \tag{3.18}$$

$$g_2(\theta|\alpha,\beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} e^{-\beta\theta} \theta^{\alpha-1}; \qquad \theta > 0, \alpha > 0, \beta > 0. \tag{3.19}$$

It may also be noted that the gamma prior $g_1(\lambda|a,b)$ and $g_2(\theta|\alpha,\beta)$ are independent and take on wide variety of shapes (prior believes of experimenter) depending on the value of hyper parameters, the joint prior pdf of λ and θ is

$$g(\lambda,\theta) = g_1(\lambda|a,b) * g_2(\theta|\alpha,\beta). \tag{3.20}$$

Combining the priors given by Equation (3.20) with likelihood given by Equation (3.12), we can easily obtain joint posterior PDF of (λ, θ) as

$$\pi(\lambda, \theta | x) = \frac{\zeta_0}{\zeta}, \tag{3.21}$$

where

$$\zeta_0 = \prod_{i=1}^{m} \frac{\theta \lambda e^{-\theta - \frac{\lambda}{x_i} + \theta e^{-\frac{\lambda}{x_i}}} \left(1 - e^{-\theta\left(1 - e^{-\frac{\lambda}{x_i}}\right)}\right)^{r_i} b^a \beta^\alpha e^{-b\lambda - \beta\theta} \lambda^{a-1} \theta^{\alpha-1}}{\Gamma(a)\Gamma(\alpha)(1 - e^{-\theta})^{r_i+1} {x_i}^2}$$

and

$$\zeta = \int_0^\infty \int_0^\infty \zeta_0 d\lambda d\theta.$$

Hence, the respective marginal posterior PDF's of λ and θ are given by

$$\pi_1(\lambda | x, r) = \int_0^\infty \frac{\zeta_0}{\zeta} d\theta, \tag{3.22}$$

$$\pi_2(\theta | x, r) = \int_0^\infty \frac{\zeta_0}{\zeta} d\lambda. \tag{3.23}$$

Further, in this context of loss function is the very essential element of the parameter estimation problem. SELF is very frequently used loss function and the weakness of this loss function is symmetric and put on equal weight to o.e. and u.e. of the same magnitude. Also, considered asymmetric loss function is GELF and LINEX are discussed in Chapter 1, Section 1.8.

Expressions for the Bayesian estimators $\left(\hat{\lambda}_S, \hat{\theta}_S\right)$, $\left(\hat{\lambda}_L, \hat{\theta}_L\right)$ and $\left(\hat{\lambda}_G, \hat{\theta}_G\right)$ for (λ, θ) under SELF, LINEX and GELF respectively can be given as

$$\left.\begin{aligned} \hat{\lambda}_S &= \int_0^\infty \lambda \pi_1(\lambda|x,r)d\lambda, \\ \hat{\theta}_S &= \int_0^\infty \theta \pi_2(\theta|x,r)d\theta, \end{aligned}\right\} \tag{3.24}$$

$$\left.\begin{aligned} \hat{\lambda}_L &= -\frac{1}{\delta}\ln\left(\int_0^\infty e^{-\delta\lambda}\pi_1(\lambda|x,r)d\lambda\right), \\ \hat{\theta}_L &= -\frac{1}{\delta}\ln\left(\int_0^\infty e^{-\delta\theta}\pi_2(\theta|x,r)d\theta\right), \end{aligned}\right\} \tag{3.25}$$

$$\left.\begin{aligned} \hat{\lambda}_G &= \left(\int_0^\infty \lambda^{-\delta}\pi_1(\lambda|x,r)d\lambda\right)^{-\frac{1}{\delta}}, \\ \hat{\theta}_G &= \left(\int_0^\infty \theta^{-\delta}\pi_2(\theta|x,r)d\theta\right)^{-\frac{1}{\delta}}. \end{aligned}\right\} \tag{3.26}$$

Here, δ is the shape parameter of loss function. Substituting the posterior PDF from Equation (3.22) and (3.23) in Equations (3.24), (3.25) and (3.26) respectively and then simplifying, we get the Bayesian estimators $\left(\hat{\lambda}_S, \hat{\theta}_S\right)$, $\left(\hat{\lambda}_L, \hat{\theta}_L\right)$ and $\left(\hat{\lambda}_G, \hat{\theta}_G\right)$ of (λ, θ). It may also be noted that the above integrals involved in the expressions for the Bayesian estimators $\left(\hat{\lambda}_S, \hat{\theta}_S\right)$, $\left(\hat{\lambda}_L, \hat{\theta}_L\right)$ and $\left(\hat{\lambda}_G, \hat{\theta}_G\right)$ are not possible to reduce in closed form. Therefore, we propose the use of numerical method for obtaining the estimates. We have used MCMC method. For this, we proceed by generating observations from posteriors Equations (3.22) and (3.23) respectively. These posteriors does not follow any standard form density, since with help of Metropolis and Ulam (1949), we use M-H algorithm to generate sample observations from each of these posterior distributions as, see Gelman et al. (2013)

$$\pi_1^*(\lambda|\theta,x,r) \propto \lambda^{m+a-1} e^{-\lambda\sum_{i=1}^m\left(\frac{1}{x_i}+b\right)} e^{\theta\sum_{i=1}^m e^{-\frac{\lambda}{x_i}}} \prod_{i=1}^m \left(1-e^{-\theta\left(1-e^{-\frac{\lambda}{x_i}}\right)}\right)^{r_i}, \tag{3.27}$$

$$\pi_2^*(\theta|\lambda,x,r) \propto \frac{\theta^{m+\alpha-1}e^{-\theta(m+\beta)}e^{\theta\sum_{i=1}^{m}e^{-\frac{\lambda}{x_i}}}\prod_{i=1}^{m}\left(1-e^{-\theta\left(1-e^{-\frac{\lambda}{x_i}}\right)}\right)^{r_i}}{(1-e^{-\theta})^{\sum_{i=1}^{m}r_i+m}}, \tag{3.28}$$

respectively. For detailed see Aggarwala and Balakrishnan (1998), Louzada-Neto et al. (2011), Wu and Chang (2002)

3.4.2 E-Bayesian Estimation

The prior distribution of λ and θ are $g_1(\lambda|a,b)$ and $g_2(\theta|\alpha,\beta)$ respectively, which are decreasing function of $\lambda,\ \theta$ with respective hyper parameter (a,b) and (α,β), for more detailed see Han (2007). The derivative of $g_1(\lambda|a,b)$ and $g_2(\theta|\alpha,\beta)$ with respect to λ,θ is

$$\frac{d[g_1(\lambda|a,b)]}{d\lambda} = \frac{b^a\lambda^{a-2}e^{-b\lambda}}{\Gamma(a)}\left[(a-1)-b\lambda\right], \tag{3.29}$$

$$\frac{d[g_2(\theta|\alpha,\beta)]}{d\theta} = \frac{\beta^\alpha\theta^{\alpha-2}e^{-\beta\theta}}{\Gamma(\alpha)}\left[(\alpha-1)-\beta\theta\right], \tag{3.30}$$

where $(a,b)>0,(\alpha,\beta)>0$ and $(\lambda,\theta)>0$. From above Equation (3.29) and Equation (3.30), it is clear that for $0<(a,\alpha)<1,(b,\beta)>0$, and since $g_1(\lambda|a,b)$, $g_2(\theta|\alpha,\beta)$ is decreasing function of λ,θ. It also observed for given $0<(a,\alpha)<1$ and $(b,\beta)>0$ are, the very less probability in the tail of the gamma density function. According to Berger (2013), the thinner tailed prior distribution often reduces the robustness of Bayesian estimate. Since, the value of (b,β) should not be larger than a given upper bound (c,γ), where $(c,\gamma)>0$ is a given upper bound. Therefore, the hyper parameter (a,b) and (α,β) should be selected with the restriction of $0<(a,\alpha)<1$ and $0<b<c$, $0<\beta<\gamma$ (where constant (c,γ) would be considered later in simulation study). Then we are obtained the E-Bayesian estimate of λ and θ based on three different distribution of the hyper parameters (a,b) and (α,β). These distributions are used to investigate the influence of the different prior distributions on the E-Bayesian estimation of λ

and θ. The following distribution of hyper parameter a,b are used for λ

$$\left.\begin{aligned}
\pi_{11}(a,b) &= \frac{1}{c(B(u,v))}a^{(u-1)}(1-a)^{(v-1)}; \qquad 0<a<1; 0<b<c,\\
\pi_{12}(a,b) &= \frac{2(c-b)}{c^2B(u,v)}a^{(u-1)}(1-a)^{(v-1)}; \qquad 0<a<1; 0<b<c,\\
\pi_{13}(a,b) &= \frac{2b}{c^2B(u,v)}a^{(u-1)}(1-a)^{(v-1)}; \qquad 0<a<1; 0<b<c.
\end{aligned}\right\} \tag{3.31}$$

We may also notice from above Equations (3.31), if $u>1$, $v>1$ then $\pi_{1i}(a,b)\to 0$ as $a\to 0$ or $a\to 1; i=1,2,3$. But $0<u<1$, then $\pi_{1i}(a,b)\to\infty$ as $a\to 0$, and if $0<v<1$, then $\pi_{1i}(a,b)\to\infty$ as $a\to 1; i=1,2,3$. Therefore, $u>1$, $v>1$ then $\pi_{1i}(a,b)$ has unique optimal solutions for each $i=1,2,3$. Also, distribution of hyper parameter α,β are used for θ

$$\left.\begin{aligned}
\pi_{21}(\alpha,\beta) &= \frac{1}{\gamma B(u_1,v_1)}\alpha^{(u_1-1)}(1-\alpha)^{(v_1-1)}; \qquad 0<\alpha<1; 0<\beta<\gamma,\\
\pi_{22}(\alpha,\beta) &= \frac{2(\gamma-\beta)}{\gamma^2B(u_1,v_1)}\alpha^{(u_1-1)}(1-\alpha)^{(v_1-1)}; \qquad 0<\alpha<1; 0<\beta<\gamma,\\
\pi_{23}(\alpha,\beta) &= \frac{2\beta}{\gamma^2B(u_1,v_1)}\alpha^{(u_1-1)}(1-\alpha)^{(v_1-1)}; \qquad 0<\alpha<1; 0<\beta<\gamma.
\end{aligned}\right\} \tag{3.32}$$

Similarly, from above Equations (3.32), if $u_1>1$, $v_1>1$ then $\pi_{2i}(\alpha,\beta)\to 0$ as $\alpha\to 0$ or $\alpha\to 1; i=1,2,3$. But $0<u_1<1$, then $\pi_{2i}(\alpha,\beta)\to\infty$ as $\alpha\to 0$, and if $0<v_1<1$, then $\pi_{2i}(\alpha,\beta)\to\infty$ as $\alpha\to 1; i=1,2,3$. Therefore, $u_1>1$, $v_1>1$ then $\pi_{2i}(\alpha,\beta)$ has unique optimal solutions for each $i=1,2,3$. Since, the above Equations (3.31) and (3.32) are said to be the prior of the hyper parameter a and b for λ; α and β for θ respectively. The corresponding E-Bayesian estimation of λ and θ under SELF, GELF and LINEX are given as follows:

$$\left.\begin{aligned}
\hat{\lambda}_{EBSi} &= \int\int_D \hat{\lambda}_S\pi_{1i}(a,b)\,da\,db; \qquad i=1,2,3; D\in\{(a,b):\ 0<a<1,\ 0<b<c\},\\
\hat{\theta}_{EBSi} &= \int\int_D \hat{\theta}_S\pi_{2i}(\alpha,\beta)\,d\alpha\,d\beta; \qquad i=1,2,3; D\in\{(\alpha,\beta):\ 0<\alpha<1,\ 0<\beta<\gamma\},
\end{aligned}\right\} \tag{3.33}$$

$$\left.\begin{aligned}\hat{\lambda}_{EBGi} &= \int\int_D \hat{\lambda}_G \pi_{1i}(a,b)dadb; \qquad i=1,2,3; D\in\{(a,b):\ 0<a<1,\ 0<b<c\},\\ \hat{\theta}_{EBGi} &= \int\int_D \hat{\theta}_G \pi_{2i}(\alpha,\beta)d\alpha d\beta; \qquad i=1,2,3; D\in\{(\alpha,\beta):\ 0<\alpha<1,\ 0<\beta<\gamma\},\end{aligned}\right\} \tag{3.34}$$

and

$$\left.\begin{aligned}\hat{\lambda}_{EBLi} &= \int\int_D \hat{\lambda}_L \pi_{1i}(a,b)dadb; \qquad i=1,2,3; D\in\{(a,b):\ 0<a<1,\ 0<b<c\},\\ \hat{\theta}_{EBLi} &= \int\int_D \hat{\lambda}_L \pi_{2i}(a,b)dadb; \qquad i=1,2,3; D\in\{(\alpha,\beta):\ 0<\alpha<1,\ 0<\beta<\gamma\}.\end{aligned}\right\} \tag{3.35}$$

We observe the above Equations (3.33), (3.34) and (3.35) in integrals form of E-Bayesian estimate under SELF, GELF and LINEX loss functions are not possible to get the solution. Therefore, we perform the computational method Markov Chain Monte Carlo (MCMC) through R software and get the estimates of E-Bayesian under SELF, GELF and LINEX (see Zellner (1994) and Zellner (1986a)). In the next subsection, we deal with the MCMC algorithm.

3.4.3 The MCMC Algorithm

We now generate sample observation from $\pi_1^*(\lambda|\theta,x,r)$ and $\pi_2^*(\theta|\lambda,x,r)$, and taking normal distribution, $N\left(\hat{\lambda}_M, Var\left(\hat{\lambda}_M\right)\right)$ and $N\left(\hat{\theta}_M, Var\left(\hat{\theta}_M\right)\right)$ as proposal density respectively. The following steps of the algorithm is given as

(i) Set the initial guess of λ and θ say $\lambda_0=\hat{\lambda}_M$ and $\theta_0=\hat{\theta}_M$

(ii) Set $i=1$

(iii) Generate a candidate point λ_i^* and θ_i^* from proposal distribution $\Phi_1 \sim N\left(\hat{\lambda}_M, Var\left(\hat{\lambda}_M\right)\right)$ and $\Phi_2 \sim N\left(\hat{\theta}_M, Var\left(\hat{\theta}_M\right)\right)$ respectively and take a point $\mathbf{x}$ from uniform distribution $U(0,1)$.
Let $\tau_1\left(\lambda_i^{(i-1)},\lambda^*\right) = min\left(\frac{\pi_1^*(\lambda_1^*|\lambda_{(i-1)},x,r)\Phi_1(\lambda^{i-1})}{\pi_1^*(\lambda^*|\lambda_{i-1},x,r)\Phi_1(\lambda^*)},1\right)$,

$\tau_2\left(\theta_i^{(i-1)}, \theta^*\right) = min\left(\frac{\pi_2^*(\theta_1^*|\theta_{(i-1)},x,r)\Phi_2(\theta^{i-1})}{\pi_2^*(\theta^*|\theta_{i-1},x,r)\Phi_2(\theta^*)}, 1\right)$

Set $\lambda^{(i)} = \lambda^*$, $\theta^{(i)} = \theta^*$ if $u \leq \tau_1\left(\lambda_i^{(i-1)}, \lambda^*\right)$, $u \leq \tau_2\left(\theta_i^{(i-1)}, \theta^*\right)$ otherwise set $\lambda^{(i)} = \lambda^{(i-1)}$, $\theta^{(i)} = \theta^{(i-1)}$

(iv) Set $i = i+1$

(v) Repeat steps (ii)-(iv) for sufficiently large number of time i.e., N sufficiently large number. After the convergence of chain of observations, we have $\lambda_1, \lambda_2, \lambda_3, \cdots, \lambda_N$ and $\theta_1, \theta_2, \theta_3, \cdots, \theta_N$.

(vi) Obtain the Bayesian estimates of λ and θ under GELF and LINEX as $\hat{\lambda}_G = \left[\frac{1}{N-N_0}\sum_{i=1}^{N-N_0}\lambda_i^{-\delta}\right]^{-\frac{1}{\delta}}$, $\hat{\theta}_G = \left[\frac{1}{N-N_0}\sum_{i=1}^{N-N_0}\theta_i^{-\delta}\right]^{-\frac{1}{\delta}}$ and $\hat{\lambda}_L = -\frac{1}{\delta}\ln\left[\frac{1}{N-N_0}\sum_{i=1}^{N-N_0}e^{-\delta\lambda_i}\right]$, $\hat{\theta}_L = -\frac{1}{\delta}\ln\left[\frac{1}{N-N_0}\sum_{i=1}^{N-N_0}e^{-\delta\theta_i}\right]$, where, N_0 is the burn-in-period of Markov Chain. When substituting δ equal to -1 in step (vi), of $\hat{\lambda}_G, \hat{\theta}_G$, we get Bayesian estimates of λ and θ under SELF.

(vii) The E-Bayesian estimates of λ and θ under different loss functions, firstly we generate a and b from Beta distribution and Uniform distribution for the values of prior parameters (u, v) and $(0, c)$ respectively as stated Equation (3.31). Similarly, generate α and β from Beta distribution and Uniform distribution for the values of prior parameters (u_1, v_1) and $(0, \gamma)$ respectively as stated Equation (3.32). Using step (vi) with above Equations (3.31), (3.32) to get E-Bayesian estimates of λ and θ under SELF, GELF and LINEX respectively.

3.4.4 Bayesian Intervals

In this subsection, we obtain Bayesian credible and HPD intervals of λ and θ, which is analog of a frequentest method of CIs. We use the sample λ and θ (say $\lambda_{[1]}, \lambda_{[2]}, \lambda_{[3]}, \cdots, \lambda_{[N]}$) and $(\theta_{[1]}, \theta_{[2]}, \theta_{[3]}, \cdots, \theta_{[N]})$ obtained from posterior distribution through the MCMC method in the previous subsection and apply the algorithm of Wu and Chang (2002) to get Bayesian credible and HPD intervals for λ and θ. In this algorithm, we first order the obtained sample observations as $(\lambda_{[1]} < \lambda_{[2]} < \lambda_{[3]} < \cdots < \lambda_{[N]})$ and $(\theta_{[1]} < \theta_{[2]} < \theta_{[3]} < \cdots < \theta_{[N]})$ and then

(i) The $100(1-\psi)\%$ Bayesian credible interval of λ and θ are given as $(\lambda_{[(\psi/2)N]}, \lambda_{[(1-\psi/2)N]})$ and $(\theta_{[(\psi/2)N]}, \theta_{[(1-\psi/2)N]})$; where $[(\psi/2)N]$ and $[(1-\psi/2)N]$ are integer parts of $[(\psi/2)N]$ and $[(1-\psi/2)N]$ respectively.

(ii) For HPD of λ and θ, we first obtain all $100(1-\psi)\%$ credible intervals given as

$$(\lambda_{[i]}, \lambda_{[i+(1-\psi/2)N]}); \qquad i = 1,2,3,\cdots,\psi N;$$

$$(\theta_{[i]}, \theta_{[i+(1-\psi/2)N]}); \qquad i = 1,2,3,\cdots,\psi N;$$

along with their corresponding lengths

$$L(\lambda)_i = \lambda_{[i+(1-\psi/2)N]} - \lambda_{[i]}; \qquad i = 1,2,3,\cdots,\psi N;$$

$$L(\theta)_i = \theta_{[i+(1-\psi/2)N]} - \theta_{[i]}; \qquad i = 1,2,3,\cdots,\psi N;$$

and thereafter pick up the interval of λ and θ of which have smallest length $L(\lambda)_i$ and $L(\theta)_i$ respectively.

3.5 Monte Carlo Simulation Study and Comparison of Estimators

We are obtaining some numerical illustrations based on MC simulation study. Because, analytically the Bayesian and E-Bayesian estimators are not obtained in closed form. Therefore, we need to simulate propose estimators under PT-II CBR samples from PIED. The Balakrishnan and Sandhu (1995)'s algorithm has been used for simulation under PT-II CBR samples. Then, we also compare the various estimators computed in Sections 3.3 and 3.4 under PT II-CBRs. The set of estimators $\left(\hat{\lambda}_M, \hat{\theta}_M\right)$, $\left(\hat{\lambda}_S, \hat{\theta}_S\right)$, $\left(\hat{\lambda}_G, \hat{\theta}_G\right)$ and $\left(\hat{\lambda}_L, \hat{\theta}_L\right)$ denote the ML estimators and Bayesian estimators of the parameters (λ, θ) under SELF, GELF and LINEX respectively, while $\left(\hat{\lambda}_{EBSi},\hat{\theta}_{EBSi}\right)$, $\left(\hat{\lambda}_{EBGi},\hat{\theta}_{EBGi}\right)$ and $\left(\hat{\lambda}_{EBLi},\hat{\theta}_{EBLi}\right)$ with $i=1,2,3$ are corresponding E-Bayesian estimators under SELF, respectively. Now, we are comparing the estimators obtained under symmetric and asymmetric loss functions with corresponding Bayesian and E-Bayesian estimators respectively. The comparisons are based on the simulated risks under SELF, GELF and LINEX. It may also be mentioned here that the exact expressions for the risks can not be computed because the estimators are not found in closed form. Therefore, the risks of the estimators are estimated on the basis of MC simulation study of 10000 samples. It may also be cleared that the risks of the estimators will depend on values of $n,m,p,\lambda,\theta,\gamma$, δ and c.

In this study the effect of variation in the value of the combination of total sample size n with different size of effective sample size m, we have obtained the simulated risks for $n=20, m=12\,[2]\,4$; $n=30, m=18\,[3]\,6$ and $n=40, m=24\,[4]\,8$ and with loss parameter $\delta=+0.1$, (o.e. is more serious than u.e.), $\delta=-0.1$ (u.e. is more serious than o.e.), $p=0.05$ (probability of removals). The selection of the hyper parameter based on the prior distribution setting, here we are choosing the prior mean as a true parameter value and prior variance is $\sigma^2=0.9$. Then the hyper parameter $a=\frac{\lambda^2}{\sigma^2}$ and $b=\frac{\lambda}{\sigma^2}$, similarly $\alpha=\frac{\theta^2}{\sigma^2}$ and $\beta=\frac{\theta}{\sigma^2}$ and hyper prior parameter is $u=a,\ v=b,\ c=4$ and $u_1=\alpha,\ v_1=\beta,\quad \gamma=3$. Using PT-II CBR samples and obtained simulated risks for the estimators of λ and θ under SELF, GELF and LINEX have been obtained for selected values of $n,m,p,\lambda,\theta,\gamma$, δ and c. The results are summarized in Table (3.2), Table (3.4), Table (3.3) and Table (3.5) respectively.

The computations in Table (3.2) and Table (3.3) show that ML estimates, Bayesian and E-Bayesian estimates of parameter λ and θ based on SELF, GELF and LINEX. The estimated risks of the different estimators are compared on SELF. Table (3.2) shows that the O.e. is more serious than u.e. cases i.e, for $\delta>0$ while Table (3.3) represents that for u.e. is more serious than o.e. cases i.e, for $\delta<0$ of $\left(\hat{\lambda}_G,\hat{\theta}_G\right)$ and $\left(\hat{\lambda}_L,\hat{\theta}_L\right)$; $\left(\hat{\lambda}_{EBGi},\hat{\theta}_{EBGi}\right)$ and $\left(\hat{\lambda}_{EBLi},\hat{\theta}_{EBLi}\right)$ with $i=1,2,3$ are corresponding E-Bayesian estimators. We observed from Table (3.2), Table (3.3) that estimated risks of Bayesian and E-Bayesian estimators decrease as effective sample size m (and fixed n) increases. Generally, in most of the cases, risks of the proposed E-Bayesian estimator of λ and θ i.e, $\left(\hat{\lambda}_{EBL3},\hat{\theta}_{EBL3}\right)$ has minimum risks as compared to other competitive estimators of λ and θ. Also, in Table (3.2) and Table (3.3) show average Bayesian and E-Bayesian (in parenthesis) estimate, average length of CI and HPD intervals. The Average length of CI and HPD interval of λ and θ are also decreases when m increases.

However, in Table (3.4) shows that the risks of estimators of λ and θ under GELF and LINEX for $\delta > 0$. Under both losses we show that the proposed E-Bayesian estimator $\left(\hat{\lambda}_{EBS3}, \hat{\theta}_{EBS3}\right)$ has minimum risk as compared to other competitive estimators and the trend of risks of the estimators of λ and θ have similar trend as previous tables. But Table (3.5) shows risks of estimators of λ and θ under GELF and LINEX for $\delta < 0$. In this table we also found that the proposed E-Bayesian estimator $\left(\hat{\lambda}_{EBL3}, \hat{\theta}_{EBL3}\right)$ perform better than other estimators. For fixed n, when the effective sample size m increases risks of Bayesian and E-Bayesian estimators decreases.

3.6 An application to Survival of Multiple Myeloma Patients Data

Here, we consider a Multiple myeloma patients data set from Collett (2014). The observed data of survival time (in months) of multiple myeloma patients are 13, 52, 6, 40, 10, 7, 66, 10, 10, 14, 16, 4, 65, 5, 11, 10, 15, 5,76, 56, 88, 24, 51, 4, 40, 8, 18, 5, 16, 50, 40, 1, 36, 5, 10, 91, 18, 1, 18, 6, 1, 23, 15, 18, 12, 12, 17, 3, which are related to 48 patients, all of whom are aged between 50 to 80 years. Suppose the survival time of multiple myeloma patients who have a malignant disease characterized by the accumulation of abnormal plasma cell, a type of white blood cell, in the bone marrow can be modeled by PIED as a lifetime model.

Goodness of Fit Tests

The χ^2 goodness of fit and the K-S test to check whether PIED has properly accommodate the data. The simultaneous optimal solution of λ and θ are 2.575034 and 3.872177 respectively, which is verified by Figure (3.3). Now we want to test the null hypothesis that the distribution function $F(x)$ from which the data came PIED with $\hat{\lambda}$ = 2.575034 and $\hat{\theta}$ = 3.872177 respectively. Thus, $F(x)$ is completely specified.

The χ^2 test

The random sample of size 48 is drawn from a population with unknown CDF $F_0(x)$. We wish to test the null hypothesis

$$H_0 : F_0(x) = F(x),$$

$$H_1 : F_0(x) \neq F(x).$$

Let us start with six intervals $(0,5], (5,8], (8,14], (14,23], (23,53]$ and $(53,100]$ with equal bins. The sample size of each interval is $Y_1 = 6,\ Y_2 = 7,\ Y_3 = 10,\ Y_4 = 10,\ Y_5 = 9$, and $Y_6 = 6$ respectively. Corresponding probabilities are $p(Y_1) = 0.19366,\ p(Y_2) = 0.13691,\ p(Y_3) = 0.18103,\ p(Y_4) = 0.14482,\ p(Y_5) = 0.17226$ and $p(Y_6) = 0.07559$. The calculated value of $\chi^2_{cal} = 4.34604$ has less than tabulated value of $\chi^2_{3,0.05} = 7.81473$. Sine, we cannot reject H_0 at $\alpha\%$ level of significance. Thus $F(x)$ is suitable for the data set. But the χ^2 test is essentially applicable for large samples. Although it is also observed that the latter treats individual observations directly, whereas the former discretized the data and sometimes loses information through grouping. Therefore, the K-S test is applicable even in the case of very small samples as well as large samples.

The K–S test

This test assumes continuous of the distribution function, to check difference between $F_n(x)$ and $F(x)$. Since, to test

$$H_0 : F_n(x) = F(x),$$

$$H_1 : F_n(x) \neq F(x),$$

where $F_n(x)$ is the sample (empirical) distribution function, $F(x)$ is specified for all x. The test statistic

$$D_n = sup|F_n(x) - F(x)|,$$

is less then tabulated value of K-S distance $D_{n,\alpha}$ then accept H_0. For complete data set K-S distance and p-value are 0.125 and 0.8475, respectively. Also for various censoring schemes under PT-II CBRs, we calculate the K-S distance and corresponding p-value, see in Table (3.1). Other hand, the D_n statistic is used to obtain the confidence bands on $F_n(x)$ for all x, where $F_n(x)$ is a consistent estimator for CDF $F(x)$. The number $D_{n,\alpha}$ is obtain from the K-S table (Standard), such that

$$P[sup|F_n(x) - F(x)| < D_{n,\alpha}] = 1 - \alpha,$$

where, $0 \leq F(x) \leq 1 \; \forall x$. Thus we define

$$L_n(x) = max[F_n(x) - D_{n,\alpha}, 0],$$

and

$$U_n(x) = min[F_n(x) + D_{n,\alpha}, 1],$$

where $L_n(x)$ and $U_n(x)$ are lower and upper confidence band for the cdf $F(x)$, with $(1-\alpha)\%$ confidence coefficient. Of course, the $F(x)$ lies completely within the limits if and only if the hypothesis cannot be rejected at $\alpha\%$ level of significance. The K-S bound for various scheme are shown in the Figure (3.7).

TABLE 3.1: Real data analysis of various schemes to obtain estimates of parameter, log-L, K-S distance, p-value.

$\hat{\lambda}$	$\hat{\theta}$	log-L	K-S	p-value	Sample
2.677374	4.60836	120.5736	0.17857	0.76364	1,3,4,5,6,8,10,10,11,12,12,15,16,16,17,18,18,18,23,24,40,40,51,52,56,65,66,76
2.749227	3.828369	101.981	0.16667	0.8928	1,1,4,5,7,10,10,10,12,12,13,14,15,16,17,18,24,40,40,40,50,51,56,65
2.848905	3.543231	83.73955	0.15789	0.9718	1,1,4,5,8,10,11,12,16,18,18,24,36,40,40,51,52,56,65
2.275538	3.984181	54.4534	0.21429	0.9048	1,4,5,8,10,10,12,15,16,17,18,24,36,40
2.112439	3.18371	35.77236	0.4	0.4005	1,3,6,10,10,12,14,16,18,18

Also, we considered, a graphical method based on TTT plot as a crude indicator see Aarset (1987). The empirical TTT is given as

$$\frac{S_r}{S_n} = \frac{\sum_{i=1}^{r} x_{(i)} + (n-r)x_{(r)}}{\sum_{i=1}^{n} x_{(i)}},$$

where $r = 1, 2, \cdots, n$ and $x_{(r)}$ is the order statistics of the sample. Figure (3.1), represents the TTT plot for given data set and it indicate the increasing then decreasing failure rate functions along with PDF/CDF plot. Figure (3.2), represents the PP plot, sample QQ plot, and KMP plot i.e., Kaplan–Meier plot respectively, which can be suitable to PIED. Figure (3.3), represents the K-S plot, hazard plot and Likelihood plots, Contour plot, Contour3D plot respectively.

Data Analysis

Figure (3.4), shows the PDF in first column and CDF in second column, Figure (3.5) shows the P-P plot in first column and Q-Q plot in second column, Figure (3.6) shows the TTT plot in first column and KM plot in second column, it all for different combinations $n = 48, m = 10, 19, 24, 28$. These figures are helpful for showing which features of the data sets are well captured by the PIED model. We may also see from Figure (3.7) at several combination ($n = 48, m = 10, 19, 24, 28$) of EDF (empirical distribution function) plot with K-S bound and hazard plot. From all the graphs, we have not shown the major discrepancies between the sampled distribution (Observed values) and PIED. Further, Table (3.6) shows various combinations of censoring schemes (n,m) i.e., $(48,28),(48,24),(48,19),(48,14),(48,10)$ under PT-II CBR for survival of multiple myeloma patients data set with arbitrary choice of probability of removal $p = 0.05$. Based on all these combinations (n,m) of PT-II CBR, we obtained ML, Bayesian, E-Bayesian estimates of λ, θ under SELF, GELF and LINEX loss function, when $\delta > 0$ and $\delta < 0$ are presented in Tables ((3.8), (3.8)) and Tables ((3.9), (3.10)) with different set of values $(c,\gamma) = (3,2);(4,3)$. We also observed that for all combination of censoring schemes ML, Bayesian and E-Bayesian estimates of λ, θ under SELF, GELF and LINEX loss functions always lies in CI and HPD interval. Even though the each an every combination of censoring scheme for Table (3.6), the length of HPD always less than length of CI, see Tables (3.7), (3.8), (3.9) and Table (3.10) respectively. We now discuss some quantiles and estimate of λ, θ results obtained from PIED model with different samples, which are shown in Table (3.11) and Table (3.12) for $\delta = 0.1$ and $\delta = -0.1$ respectively. It is very interesting to note that through E-Bayesian approach covers the more survival time of myeloma patients in respect of two existing approach i.e. ML method and Bayesian method.

3.7 Conclusion

In this chapter, firstly we have studied on E-Bayesian method to compare with Bayesian estimators for both unknown parameters of PIED under PT-II CBRs. On the other hand, the risk of the E-Bayesian and Bayesian estimators of λ and θ are compared under SELF, GELF and LINEX. Generally, we found that the estimated risk of the E-Bayesian estimate of λ and θ have minimum. Therefore, the simulated results showed that the E-Bayesian estimation method is more efficient and better to perform than Bayesian estimation. Beside this, we have shown the interest with application to survival time of multiple myeloma patients data and applying E-Bayesian inferential procedures for the PIED as underline distribution with PT-II CBRs.

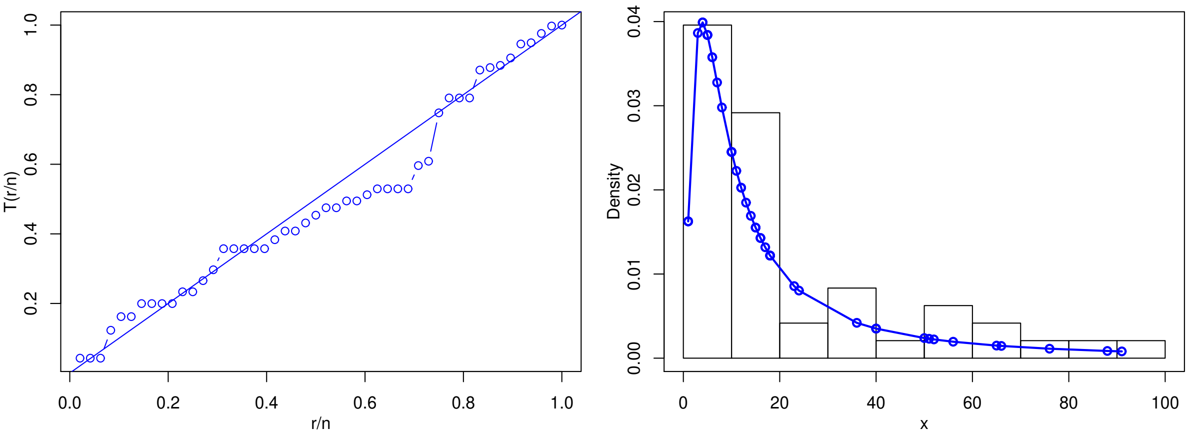

FIGURE 3.1: The Survival time of multiple myeloma patients data, Left panel: TTT plot; Right panel: PDF plot

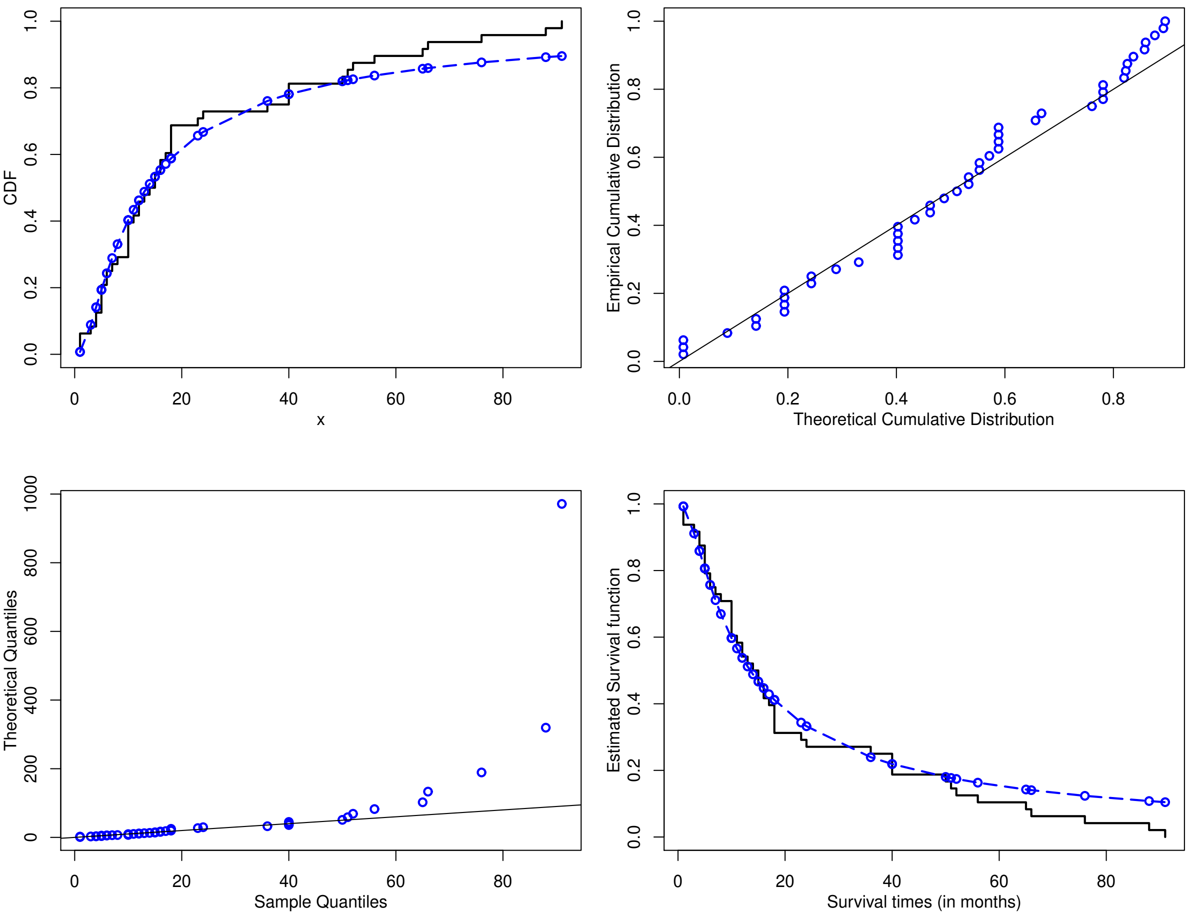

FIGURE 3.2: The survival time of multiple myeloma patients data, Left panel: CDF plot and Q-Q plot, Right panel: P-P plot and KMP plot

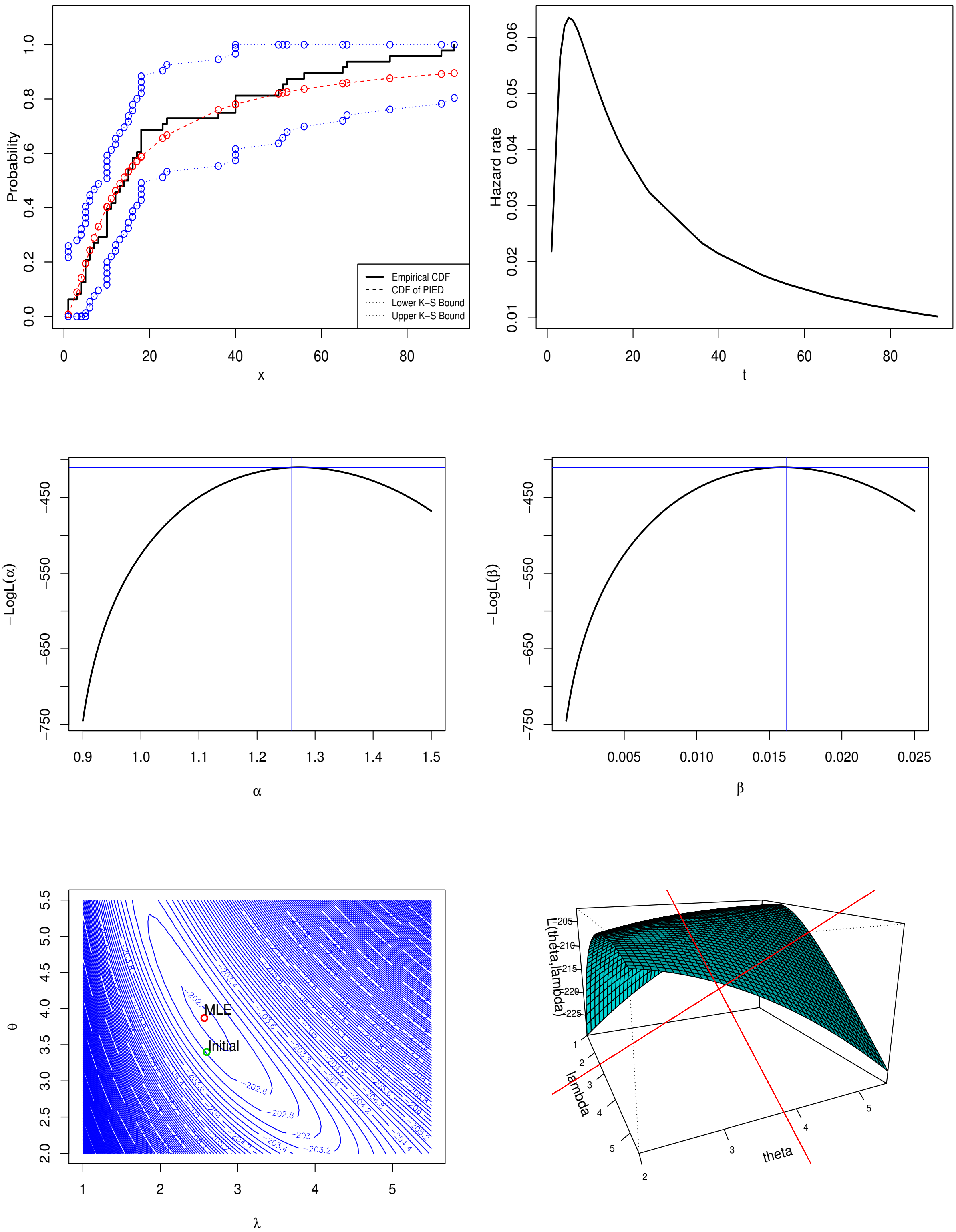

FIGURE 3.3: K-S plot, hazard plot and Likelihood plots, Contour plot, Contour3D plot shows the ML estimates of λ and θ based on the survival time of multiple myeloma patients data.

TABLE 3.2: Risks and different estimators, CI and HPD interval for parameters λ and θ under SELF for fixed $\lambda = 1.1$, $\theta = 0.8$, $\delta = 0.1, p = 0.05, c = 4, \gamma = 3$.

(n,m)		$\hat{\lambda}_S$	$\hat{\lambda}_G$	$\hat{\lambda}_L$	$\hat{\lambda}_{ML}$	CI/HPD	$\hat{\theta}_S$	$\hat{\theta}_G$	$\hat{\theta}_L$	$\hat{\theta}_{ML}$	CI/HPD
(20,12)	$\hat{R}_S(.)$	0.07569	0.07580	0.07569	0.07573	(0.41713, 1.54596)	0.30367	0.35083	0.30565	0.31641	(0.07498, 0.40008)
	$B(.)$	(0.98338)	(0.98241)	(0.98304)	(0.98155)		(0.24899)	(0.20788)	(0.24719)	(0.23753)	
	$\hat{R}_{EBS1}(.)$	0.07567	0.07578	0.07567		(0.91379, 1.05288)	0.30366	0.35081	0.30564		(0.0829, 0.41571)
	$EBS_1(.)$	(0.98341)	(0.98245)	(0.98308)			(0.24900)	(0.20790)	(0.24720)		
	$\hat{R}_{EBS2}(.)$	0.07536	0.07547	0.07535			0.30364	0.35080	0.30562		
	$EBS_2(.)$	(0.98370)	(0.98273)	(0.98337)			(0.24902)	(0.20791)	(0.24722)		
	$\hat{R}_{EBS3}(.)$	0.07605	0.07616	0.07605			0.30368	0.35083	0.30566		
	$EBS_3(.)$	(0.98313)	(0.98216)	(0.98279)			(0.24898)	(0.20789)	(0.24718)		
(20,10)	$\hat{R}_S(.)$	0.10622	0.10650	0.10626	0.10710	(0.33668, 1.45314)	0.36819	0.41628	0.36962	0.38088	(0.05158, 0.31412)
	$B(.)$	(0.89721)	(0.89617)	(0.89687)	(0.89492)		(0.19323)	(0.15505)	(0.19205)	(0.18285)	
	$\hat{R}_{EBS1}(.)$	0.10630	0.10657	0.10633		(0.82849, 0.96591)	0.36810	0.41621	0.36954		(0.05786, 0.32853)
	$EBS_1(.)$	(0.89712)	(0.89608)	(0.89679)			(0.19330)	(0.15511)	(0.19212)		
	$\hat{R}_{EBS2}(.)$	0.10598	0.10625	0.10601			0.36798	0.41609	0.36942		
	$EBS_2(.)$	(0.89711)	(0.89608)	(0.89678)			(0.19340)	(0.15520)	(0.19222)		
	$\hat{R}_{EBS3}(.)$	0.10668	0.10695	0.10671			0.36823	0.41632	0.36966		
	$EBS_3(.)$	(0.89713)	(0.89609)	(0.89680)			(0.19320)	(0.15502)	(0.19202)		

Continued on next page

Table 3.2 – *Continued from previous page*

(n,m)		$\hat{\lambda}_S$	$\hat{\lambda}_G$	$\hat{\lambda}_L$	$\hat{\lambda}_{ML}$	CI/HPD	$\hat{\theta}_S$	$\hat{\theta}_G$	$\hat{\theta}_L$	$\hat{\theta}_{ML}$	CI/HPD
(20,8)	$\hat{R}_S(.)$	0.13512	0.13563	0.13522	0.13587	(0.25083, 1.37284)	0.42862	0.47833	0.42965	0.43943	(0.03085, 0.24338)
	$B(.)$	(0.81319)	(0.81203)	(0.81286)	(0.81183)		(0.14531)	(0.10848)	(0.14453)	(0.13711)	
	$\hat{R}_{EBS1}(.)$	0.13507	0.13559	0.13517		(0.74408, 0.88234)	0.42863	0.47834	0.42966		(0.03354, 0.25507)
	$EBS_1(.)$	(0.81317)	(0.81201)	(0.81284)			(0.14530)	(0.10847)	(0.14452)		
	$\hat{R}_{EBS2}(.)$	0.13511	0.13562	0.13521			0.42869	0.47838	0.42971		
	$EBS_2(.)$	(0.81297)	(0.81181)	(0.81263)			(0.14526)	(0.10844)	(0.14448)		
	$\hat{R}_{EBS3}(.)$	0.13509	0.13560	0.13519			0.42858	0.47830	0.42961		
	$EBS_3(.)$	(0.81338)	(0.81222)	(0.81304)			(0.14535)	(0.10850)	(0.14456)		
(20,6)	$\hat{R}_S(.)$	0.19109	0.19194	0.19126	0.19269	(0.15097, 1.28286)	0.48536	0.53862	0.48606	0.49300	(0.012782, 0.182939)
	$B(.)$	(0.71886)	(0.71752)	(0.71851)	(0.71692)		(0.10333)	(0.06615)	(0.10282)	(0.09786)	
	$\hat{R}_{EBS1}(.)$	0.19115	0.19200	0.19132		(0.64896, 0.78815)	0.48538	0.53864	0.48608		(0.005229, 0.18151)
	$EBS_1(.)$	(0.71873)	(0.71739)	(0.71838)			(0.10333)	(0.06615)	(0.10283)		
	$\hat{R}_{EBS2}(.)$	0.19096	0.19181	0.19112			0.48535	0.53861	0.48605		
	$EBS_2(.)$	(0.71873)	(0.71739)	(0.71838)			(0.10333)	(0.06615)	(0.10283)		
	$\hat{R}_{EBS3}(.)$	0.19139	0.19224	0.19156			0.48541	0.53866	0.48611		
	$EBS_3(.)$	(0.71902)	(0.71769)	(0.71867)			(0.10329)	(0.06611)	(0.10279)		

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Table 3.2 – *Continued from previous page*

(n,m)		$\hat{\lambda}_S$	$\hat{\lambda}_G$	$\hat{\lambda}_L$	$\hat{\lambda}_{ML}$	CI/HPD	$\hat{\theta}_S$	$\hat{\theta}_G$	$\hat{\theta}_L$	$\hat{\theta}_{ML}$	CI/HPD
(20,4)	$\hat{R}_S(.)$	0.28077	0.28223	0.28104	0.28269	(0.02582, 1.17301)	0.54067	0.59060	0.54109	0.54392	(-0.00233, 0.12731)
	$B(.)$	(0.60146)	(0.59982)	(0.60110)	(0.59941)		(0.06470)	(0.03152)	(0.06441)	(0.06249)	
	$\hat{R}_{EBS1}(.)$	0.28090	0.28236	0.28117		(0.53087, 0.67187)	0.54068	0.59060	0.54110		(0.00004, 0.1212)
	$EBS_1(.)$	(0.60135)	(0.59972)	(0.60100)			(0.06469)	(0.03152)	(0.06441)		
	$\hat{R}_{EBS2}(.)$	0.28086	0.28232	0.28113			0.54064	0.59057	0.54106		
	$EBS_2(.)$	(0.60164)	(0.60001)	(0.60129)			(0.06472)	(0.03153)	(0.06444)		
	$\hat{R}_{EBS3}(.)$	0.28096	0.28242	0.28124			0.54073	0.59062	0.54114		
	$EBS_3(.)$	(0.60106)	(0.59943)	(0.60071)			(0.06466)	(0.03150)	(0.06438)		
(30,18)	$\hat{R}_S(.)$	0.05510	0.05521	0.05512	0.05569	(0.51424, 1.41804)	0.29228	0.31250	0.29360	0.31208	(0.10564, 0.37713)
	$B(.)$	(0.97010)	(0.96947)	(0.96989)	(0.96614)		(0.25942)	(0.24104)	(0.25820)	(0.24138)	
	$\hat{R}_{EBS1}(.)$	0.05513	0.05524	0.05515		(0.91447, 1.02564)	0.29224	0.31247	0.29357		(0.12318, 0.39602)
	$EBS_3(.)$	(0.96988)	(0.96925)	(0.96966)			(0.25945)	(0.24106)	(0.25823)		
	$\hat{R}_{EBS1}(.)$	0.05505	0.05516	0.05507			0.29233	0.31255	0.29365		
	$EBS_2(.)$	(0.96960)	(0.96897)	(0.96939)			(0.25937)	(0.24099)	(0.25815)		
	$\hat{R}_{EBS3}(.)$	0.05527	0.05539	0.05529			0.29216	0.31239	0.29349		

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Table 3.2 – *Continued from previous page*

(n,m)		$\hat{\lambda}_S$	$\hat{\lambda}_G$	$\hat{\lambda}_L$	$\hat{\lambda}_{ML}$	CI/HPD	$\hat{\theta}_S$	$\hat{\theta}_G$	$\hat{\theta}_L$	$\hat{\theta}_{ML}$	CI/HPD
	$EBS_3(.)$	(0.97015)	(0.96953)	(0.96994)			(0.25953)	(0.24113)	(0.25830)		
(30,15)	$\hat{R}_S(.)$	0.08814	0.08837	0.08819	0.08970	(0.43187, 1.32347)	0.36033	0.37980	0.36127	0.37894	(0.07605, 0.29282)
	$B(.)$	(0.88239)	(0.88172)	(0.88218)	(0.87767)		(0.19974)	(0.18374)	(0.19895)	(0.18443)	
	$\hat{R}_{EBS1}(.)$	0.08811	0.08834	0.08816		(0.82775, 0.93721)	0.36027	0.37975	0.36122		(0.09078, 0.30932)
	$EBS_1(.)$	(0.88231)	(0.88164)	(0.88211)			(0.19979)	(0.18378)	(0.19900)		
	$\hat{R}_{EBS2}(.)$	0.08826	0.08849	0.08831			0.36022	0.37970	0.36116		
	$EBS_2(.)$	(0.88237)	(0.88170)	(0.88216)			(0.19983)	(0.18382)	(0.19905)		
	$\hat{R}_{EBS3}(.)$	0.08801	0.08824	0.08806			0.36033	0.37980	0.36127		
	$EBS_3(.)$	(0.88226)	(0.88159)	(0.88206)			(0.19974)	(0.18374)	(0.19896)		
(30,12)	$\hat{R}_S(.)$	0.12723	0.12762	0.12732	0.12973	(0.34415, 1.23115)	0.42223	0.44340	0.42289	0.43857	(0.05051, 0.22506)
	$B(.)$	(0.79251)	(0.79177)	(0.79230)	(0.78765)		(0.15022)	(0.13416)	(0.14970)	(0.13776)	
	$\hat{R}_{EBS1}(.)$	0.12731	0.12771	0.12741		(0.73799, 0.84706)	0.42222	0.44340	0.42289		(0.06156, 0.23872)
	$EBS_1(.)$	(0.79248)	(0.79174)	(0.79227)			(0.15022)	(0.13416)	(0.14971)		
	$\hat{R}_{EBS2}(.)$	0.12714	0.12754	0.12723			0.42211	0.44330	0.42278		
	$EBS_2(.)$	(0.79241)	(0.79167)	(0.79220)			(0.15030)	(0.13423)	(0.14979)		
	$\hat{R}_{EBS3}(.)$	0.12753	0.12793	0.12762			0.42233	0.44349	0.42300		

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Table 3.2 – *Continued from previous page*

(n,m)		$\hat{\lambda}_S$	$\hat{\lambda}_G$	$\hat{\lambda}_L$	$\hat{\lambda}_{ML}$	CI/HPD	$\hat{\theta}_S$	$\hat{\theta}_G$	$\hat{\theta}_L$	$\hat{\theta}_{ML}$	CI/HPD
	$EBS_3(.)$	(0.79255)	(0.79181)	(0.79235)			(0.15014)	(0.13409)	(0.14962)		
(30,9)	$\hat{R}_S(.)$	0.19194	0.19256	0.19207	0.19479	(0.24596, 1.13782)	0.48073	0.50515	0.48118	0.49303	(0.02841, 0.16727)
	$B(.)$	(0.69648)	(0.69563)	(0.69627)	(0.69189)		(0.10666)	(0.08928)	(0.10633)	(0.09784)	
	$\hat{R}_{EBS1}(.)$	0.19198	0.19260	0.19211		(0.64199, 0.75135)	0.48068	0.50511	0.48114		(0.03512, 0.17785)
	$EBS_1(.)$	(0.69641)	(0.69556)	(0.69620)			(0.10669)	(0.08931)	(0.10636)		
	$\hat{R}_{EBS2}(.)$	0.19241	0.19303	0.19255			0.48056	0.50500	0.48102		
	$EBS_2(.)$	(0.69609)	(0.69556)	(0.69620)			(0.10669)	(0.08931)	(0.10636)		
	$\hat{R}_{EBS3}(.)$	0.19158	0.19220	0.19171			0.48080	0.50522	0.48126		
	$EBS_3(.)$	(0.69673)	(0.69556)	(0.69620)			(0.10669)	(0.08931)	(0.10636)		
(30,6)	$\hat{R}_S(.)$	0.28659	0.28760	0.28678	0.29050	(0.12579, 1.03098)	0.53727	0.56821	0.53755	0.54439	(0.00956, 0.11478)
	$B(.)$	(0.58245)	(0.58140)	(0.58224)	(0.57839)		(0.06701)	(0.04622)	(0.06682)	(0.06218)	
	$\hat{R}_{EBS1}(.)$	0.28653	0.28755	0.28673		(0.52707, 0.63826)	0.53727	0.56822	0.53756		(0.01005, 0.12021)
	$EBS_1(.)$	(0.58246)	(0.58141)	(0.58225)			(0.06701)	(0.04621)	(0.06682)		
	$\hat{R}_{EBS2}(.)$	0.28649	0.28750	0.28668			0.53729	0.56823	0.53757		
	$EBS_2(.)$	(0.58249)	(0.58144)	(0.58227)			(0.06700)	(0.04621)	(0.06681)		
	$\hat{R}_{EBS3}(.)$	0.28661	0.28763	0.28680			0.53726	0.56821	0.53754		
	$EBS_3(.)$	(0.58243)	(0.58144)	(0.58227)			(0.06700)	(0.04621)	(0.06681)		

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Table 3.2 – *Continued from previous page*

(n,m)		$\hat{\lambda}_S$	$\hat{\lambda}_G$	$\hat{\lambda}_L$	$\hat{\lambda}_{ML}$	CI/HPD	$\hat{\theta}_S$	$\hat{\theta}_G$	$\hat{\theta}_L$	$\hat{\theta}_{ML}$	CI/HPD
(40,24)	$\hat{R}_S(.)$	0.04457	0.04465	0.04459	0.04533	(0.58674, 1.38123)	0.28437	0.29737	0.28538	0.30786	(0.12499, 0.36536)
	$B(.)$	(0.98973)	(0.98925)	(0.98957)	(0.98398)		(0.26679)	(0.25472)	(0.26584)	(0.24518)	
	$\hat{R}_{EBS1}(.)$	0.04459	0.04467	0.04461		(0.94095, 1.03846)	0.28430	0.29730	0.28531		(0.14675, 0.38678)
	$EBS_1(.)$	(0.98969)	(0.98922)	(0.98953)			(0.26685)	(0.25479)	(0.26591)		
	$\hat{R}_{EBS2}(.)$	0.04491	0.04498	0.04492			0.28427	0.29727	0.28528		
	$EBS_2(.)$	(0.98944)	(0.98922)	(0.98953)			(0.26685)	(0.25479)	(0.26591)		
	$\hat{R}_{EBS3}(.)$	0.04434	0.04442	0.04436			0.28433	0.29733	0.28534		
	$EBS_3(.)$	(0.98994)	(0.98922)	(0.98953)			(0.26685)	(0.25479)	(0.26591)		
(40,20)	$\hat{R}_S(.)$	0.06843	0.06861	0.06848	0.07039	(0.49933, 1.27881)	0.35422	0.36643	0.35493	0.37614	(0.09132, 0.28209)
	$B(.)$	(0.89571)	(0.89521)	(0.89556)	(0.88907)		(0.20485)	(0.19468)	(0.20425)	(0.18671)	
	$\hat{R}_{EBS1}(.)$	0.06845	0.06863	0.06850		(0.84803, 0.94363)	0.35411	0.36632	0.35483		(0.10949, 0.30039)
	$EBS_1(.)$	(0.89573)	(0.89522)	(0.89557)			(0.20494)	(0.19477)	(0.20434)		
	$\hat{R}_{EBS2}(.)$	0.06832	0.06849	0.06836			0.35412	0.36633	0.35483		
	$EBS_2(.)$	(0.89617)	(0.89566)	(0.89601)			(0.20494)	(0.19476)	(0.20434)		
	$\hat{R}_{EBS3}(.)$	0.06865	0.06883	0.06869			0.35411	0.36632	0.35482		
	$EBS_3(.)$	(0.89528)	(0.89566)	(0.89601)			(0.20494)	(0.19476)	(0.20434)		

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Table 3.2 – *Continued from previous page*

(n,m)		$\hat{\lambda}_S$	$\hat{\lambda}_G$	$\hat{\lambda}_L$	$\hat{\lambda}_{ML}$	CI/HPD	$\hat{\theta}_S$	$\hat{\theta}_G$	$\hat{\theta}_L$	$\hat{\theta}_{ML}$	CI/HPD
(40,16)	$\hat{R}_S(.)$	0.11770	0.11800	0.11777	0.12114	(0.40579, 1.17501)	0.41857	0.43076	0.41907	0.43734	(0.06251, 0.21487)
	$B(.)$	(0.79740)	(0.79685)	(0.79724)	(0.79040)		(0.15304)	(0.14368)	(0.15265)	(0.13869)	
	$\hat{R}_{EBS1}(.)$	0.11771	0.11801	0.11778		(0.75031, 0.8444)	0.41857	0.43076	0.41907		(0.07666, 0.22974)
	$EBS_1(.)$	(0.79746)	(0.79691)	(0.79730)			(0.15304)	(0.14368)	(0.15265)		
	$\hat{R}_{EBS2}(.)$	0.11760	0.11790	0.11767			0.41866	0.43084	0.41916		
	$EBS_2(.)$	(0.79786)	(0.79691)	(0.79730)			(0.15304)	(0.14368)	(0.15265)		
	$\hat{R}_{EBS3}(.)$	0.11787	0.11816	0.11794			0.41848	0.43068	0.41898		
	$EBS_3(.)$	(0.79706)	(0.79651)	(0.79690)			(0.15311)	(0.14375)	(0.15272)		
(40,12)	$\hat{R}_S(.)$	0.19020	0.19068	0.19030	0.19543	(0.30012, 1.05721)	0.47822	0.49165	0.47856	0.49276	(0.03778, 0.15829)
	$B(.)$	(0.68513)	(0.68451)	(0.68498)	(0.67866)		(0.10847)	(0.09883)	(0.10822)	(0.09804)	
	$\hat{R}_{EBS1}(.)$	0.19027	0.19075	0.19038		(0.63886, 0.73133)	0.47825	0.49168	0.47859		(0.04714, 0.16906)
	$EBS_1(.)$	(0.68506)	(0.68445)	(0.68492)			(0.10844)	(0.09881)	(0.10820)		
	$\hat{R}_{EBS2}(.)$	0.19030	0.19078	0.19040			0.47828	0.49171	0.47862		
	$EBS_2(.)$	(0.68497)	(0.68435)	(0.68483)			(0.10842)	(0.09878)	(0.10818)		
	$\hat{R}_{EBS3}(.)$	0.19028	0.19076	0.19039			0.47822	0.49165	0.47856		
	$EBS_3(.)$	(0.68516)	(0.68454)	(0.68501)			(0.10847)	(0.09883)	(0.10822)		

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Table 3.2 – *Continued from previous page*

(n,m)		$\hat{\lambda}_S$	$\hat{\lambda}_G$	$\hat{\lambda}_L$	$\hat{\lambda}_{ML}$	CI/HPD	$\hat{\theta}_S$	$\hat{\theta}_G$	$\hat{\theta}_L$	$\hat{\theta}_{ML}$	CI/HPD
(40,8)	$\hat{R}_S(.)$	0.28359	0.28435	0.28374	0.28939	(0.18689, 0.9705)	0.53493	0.55352	0.53514	0.54410	(0.01665, 0.10809)
	$B(.)$	(0.58468)	(0.58391)	(0.58453)	(0.57868)		(0.06861)	(0.05603)	(0.06847)	(0.06237)	
	$\hat{R}_{EBS1}(.)$	0.28374	0.28450	0.28389		(0.53677, 0.6326)	0.53492	0.55351	0.53513		(0.02147, 0.11579)
	$EBS_1(.)$	(0.58451)	(0.58373)	(0.58435)			(0.06862)	(0.05603)	(0.06848)		
	$\hat{R}_{EBS2}(.)$	0.28389	0.28465	0.28404			0.53489	0.55349	0.53510		
	$EBS_2(.)$	(0.58456)	(0.58378)	(0.58440)			(0.06864)	(0.05605)	(0.06849)		
	$\hat{R}_{EBS3}(.)$	0.28362	0.28437	0.28376			0.53495	0.55353	0.53516		
	$EBS_3(.)$	(0.58447)	(0.58369)	(0.58431)			(0.06860)	(0.05602)	(0.06846)		

~ *CI - Confidence Interval

~ *HPD - Height Posterior Density

~ *MLE - Maximum Likelihood Estimator

~ *Bayesian estimate (in parenthesis)

TABLE 3.3: Risks and different estimators, CI and HPD interval for parameters λ and θ under SELF for fixed $\lambda = 1.1$ and $\theta = 0.8$, $\delta = -0.1$, $p = 0.05$, $c = 4$, $\gamma = 3$.

(n,m)		$\hat{\lambda}_S$	$\hat{\lambda}_G$	$\hat{\lambda}_L$	$\hat{\lambda}_{ML}$	CI/HPD	$\hat{\theta}_S$	$\hat{\theta}_G$	$\hat{\theta}_L$	$\hat{\theta}_{ML}$	CI/HPD
(20,12)	$\hat{R}_S(.)$	0.07569	0.07573	0.07570	0.07573	(0.41713, 1.54597)	0.30367	0.31311	0.30170	0.31641	(0.07498, 0.40008)
	$B(.)$	(0.98338)	(0.98306)	(0.98372)	(0.98155)		(0.24899)	(0.24048)	(0.25078)	(0.23753)	
	$\hat{R}_{EBS1}(.)$	0.07567	0.07571	0.07568		(0.91379, 1.05288)	0.30366	0.31309	0.30168		(0.0829, 0.41571)
	$EBS_1(.)$	(0.98341)	(0.98309)	(0.98375)			(0.24900)	(0.24050)	(0.25080)		
	$\hat{R}_{EBS2}(.)$	0.07536	0.07540	0.07537			0.30364	0.31307	0.30166		
	$EBS_2(.)$	(0.98370)	(0.98338)	(0.98404)			(0.24902)	(0.24052)	(0.25082)		
	$\hat{R}_{EBS3}(.)$	0.07605	0.07609	0.07606			0.30368	0.31311	0.30171		
	$EBS_3(.)$	(0.98313)	(0.98280)	(0.98346)			(0.24898)	(0.24048)	(0.25078)		
(20,10)	$\hat{R}_S(.)$	0.10622	0.10631	0.10619	0.10710	(0.33669, 1.45315)	0.36819	0.37714	0.36676	0.38088	(0.05159, 0.31412)
	$B(.)$	(0.89721)	(0.89686)	(0.89754)	(0.89492)		(0.19323)	(0.18590)	(0.19441)	(0.18285)	
	$\hat{R}_{EBS1}(.)$	0.10630	0.10639	0.10626		(0.82849, 0.96591)	0.36810	0.37705	0.36667		(0.05786, 0.32851)
	$EBS_1(.)$	(0.89712)	(0.89678)	(0.89746)			(0.19330)	(0.18597)	(0.19448)		
	$\hat{R}_{EBS3}(.)$	0.10598	0.10607	0.10595			0.36798	0.37693	0.36655		
	$EBS_2(.)$	(0.89711)	(0.89677)	(0.89745)			(0.19340)	(0.18607)	(0.19458)		
	$\hat{R}_{EBS3}(.)$	0.10668	0.10677	0.10665			0.36823	0.37718	0.36680		
	$EBS_3(.)$	(0.89713)	(0.89678)	(0.89746)			(0.19320)	(0.18587)	(0.19438)		

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Table 3.3 – *Continued from previous page*

(n,m)		$\hat{\lambda}_S$	$\hat{\lambda}_G$	$\hat{\lambda}_L$	$\hat{\lambda}_{ML}$	CI/HPD	$\hat{\theta}_S$	$\hat{\theta}_G$	$\hat{\theta}_L$	$\hat{\theta}_{ML}$	CI/HPD
(20,8)	$\hat{R}_S(.)$	0.13512	0.13529	0.13502	0.13587	(0.2508, 1.37284)	0.42862	0.43736	0.42760	0.43943	(0.03084, 0.24338)
	$B(.)$	(0.81319)	(0.81281)	(0.81353)	(0.81183)		(0.14531)	(0.13867)	(0.14610)	(0.13711)	
	$\hat{R}_{EBS1}(.)$	0.13507	0.13525	0.13497		(0.74408, 0.88233)	0.42863	0.43737	0.42761		(0.03354, 0.25507)
	$EBS_1(.)$	(0.81317)	(0.81279)	(0.81351)			(0.14530)	(0.13866)	(0.14609)		
	$\hat{R}_{EBS2}(.)$	0.13511	0.13528	0.13500			0.42869	0.43743	0.42766		
	$EBS_2(.)$	(0.81297)	(0.81258)	(0.81331)			(0.14526)	(0.13862)	(0.14605)		
	$\hat{R}_{EBS3}(.)$	0.13509	0.13526	0.13499			0.42858	0.43732	0.42756		
	$EBS_3(.)$	(0.81338)	(0.81299)	(0.81372)			(0.14535)	(0.13870)	(0.14613)		
(20,6)	$\hat{R}_S(.)$	0.19109	0.19137	0.19093	0.19269	(0.15097, 1.28286)	0.48536	0.49411	0.48466	0.49300	(0.01278, 0.18294)
	$B(.)$	(0.71886)	(0.71841)	(0.71920)	(0.71692)		(0.10333)	(0.09708)	(0.10383)	(0.09786)	
	$\hat{R}_{EBS1}(.)$	0.19115	0.19143	0.19099		(0.64896, 0.78815)	0.48538	0.49413	0.48468		(0.00523, 0.1815)
	$EBS_1(.)$	(0.71887)	(0.71843)	(0.71922)			(0.10331)	(0.09706)	(0.10382)		
	$\hat{R}_{EBS2}(.)$	0.19096	0.19124	0.19079			0.48535	0.49410	0.48465		
	$EBS_2(.)$	(0.71873)	(0.71828)	(0.71907)			(0.10333)	(0.09708)	(0.10384)		
	$\hat{R}_{EBS3}(.)$	0.19139	0.19167	0.19123			0.48541	0.49415	0.48471		
	$EBS_3(.)$	(0.71902)	(0.71858)	(0.71937)			(0.10329)	(0.09704)	(0.10379)		

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Table 3.3 – *Continued from previous page*

(n,m)		$\hat{\lambda}_S$	$\hat{\lambda}_G$	$\hat{\lambda}_L$	$\hat{\lambda}_{ML}$	CI/HPD	$\hat{\theta}_S$	$\hat{\theta}_G$	$\hat{\theta}_L$	$\hat{\theta}_{ML}$	CI/HPD
(20,4)	$\hat{R}_S(.)$	0.28077	0.28126	0.28050	0.28269	(0.02582, 1.17301)	0.54067	0.54924	0.54026	0.54392	(-0.00233, 0.12731)
	$B(.)$	(0.60146)	(0.60091)	(0.60181)	(0.59941)		(0.06470)	(0.05890)	(0.06498)	(0.06249)	
	$\hat{R}_{EBS1}(.)$	0.28090	0.28138	0.28062		(0.53086, 0.67187)	0.54068	0.54924	0.54026		(0.00003, 0.1212)
	$EBS_1(.)$	(0.60135)	(0.60081)	(0.60171)			(0.06469)	(0.05889)	(0.06498)		
	$\hat{R}_{EBS2}(.)$	0.28086	0.28134	0.28059			0.54064	0.54920	0.54022		
	$EBS_2(.)$	(0.60164)	(0.60110)	(0.60200)			(0.06472)	(0.05892)	(0.06501)		
	$\hat{R}_{EBS3}(.)$	0.28096	0.28145	0.28069			0.54073	0.54928	0.54031		
	$EBS_3(.)$	(0.60106)	(0.60052)	(0.60142)			(0.06466)	(0.05887)	(0.06495)		
(30,18)	$\hat{R}_S(.)$	0.05510	0.05513	0.05508	0.05569	(0.51424, 1.41804)	0.29228	0.29785	0.29096	0.31208	(0.10564, 0.37712)
	$B(.)$	(0.97010)	(0.96989)	(0.97031)	(0.96614)		(0.25942)	(0.25429)	(0.26064)	(0.24138)	
	$\hat{R}_{EBS1}(.)$	0.05513	0.05517	0.05511		(0.91447, 1.02564)	0.29224	0.29782	0.29093		(0.12318, 0.39602)
	$EBS_1(.)$	(0.96988)	(0.96967)	(0.97009)			(0.25945)	(0.25432)	(0.26067)		
	$\hat{R}_{EBS2}(.)$	0.05505	0.05509	0.05503			0.29233	0.29791	0.29101		
	$EBS_2(.)$	(0.96960)	(0.96939)	(0.96981)			(0.25937)	(0.25424)	(0.26060)		
	$\hat{R}_{EBS3}(.)$	0.05527	0.05531	0.05525			0.29216	0.29774	0.29085		

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Table 3.3 – *Continued from previous page*

(n,m)		$\hat{\lambda}_S$	$\hat{\lambda}_G$	$\hat{\lambda}_L$	$\hat{\lambda}_{ML}$	CI/HPD	$\hat{\theta}_S$	$\hat{\theta}_G$	$\hat{\theta}_L$	$\hat{\theta}_{ML}$	CI/HPD
	$EBS_3(.)$	(0.97015)	(0.96994)	(0.97036)			(0.25953)	(0.25439)	(0.26075)		
(30,15)	$\hat{R}_S(.)$	0.08814	0.08822	0.08809	0.08970	(0.43187, 1.32346)	0.36033	0.36555	0.35939	0.37894	(0.07605, 0.29282)
	$B(.)$	(0.88239)	(0.88217)	(0.88260)	(0.87767)		(0.19974)	(0.19541)	(0.20052)	(0.18443)	
	$\hat{R}_{EBS1}(.)$	0.08811	0.08819	0.08806		(0.82775, 0.93721)	0.36027	0.36549	0.35933		(0.09078, 0.30932)
	$EBS_1(.)$	(0.88231)	(0.88209)	(0.88252)			(0.19979)	(0.19545)	(0.20057)		
	$\hat{R}_{EBS2}(.)$	0.08826	0.08834	0.08821			0.36022	0.36544	0.35928		
	$EBS_2(.)$	(0.88237)	(0.88214)	(0.88257)			(0.19983)	(0.19550)	(0.20062)		
	$\hat{R}_{EBS3}(.)$	0.08801	0.08809	0.08796			0.36033	0.36555	0.35939		
	$EBS_3(.)$	(0.88226)	(0.88204)	(0.88247)			(0.19974)	(0.19541)	(0.20052)		
(30,12)	$\hat{R}_S(.)$	0.12723	0.12736	0.12714	0.12973	(0.34415, 1.23115)	0.42223	0.42725	0.42156	0.43857	(0.05051, 0.22501)
	$B(.)$	(0.79251)	(0.79226)	(0.79271)	(0.78765)		(0.15022)	(0.14636)	(0.15073)	(0.13776)	
	$\hat{R}_{EBS1}(.)$	0.12731	0.12745	0.12722		(0.73799, 0.84707)	0.42222	0.42725	0.42156		(0.06156, 0.23872)
	$EBS_1(.)$	(0.79248)	(0.79223)	(0.79268)			(0.15022)	(0.14636)	(0.15073)		
	$\hat{R}_{EBS2}(.)$	0.12714	0.12727	0.12705			0.42211	0.42714	0.42145		
	$EBS_2(.)$	(0.79241)	(0.79216)	(0.79261)			(0.15030)	(0.14644)	(0.15082)		
	$\hat{R}_{EBS3}(.)$	0.12753	0.12766	0.12744			0.42233	0.42736	0.42167		

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Table 3.3 – *Continued from previous page*

(n,m)		$\hat{\lambda}_S$	$\hat{\lambda}_G$	$\hat{\lambda}_L$	$\hat{\lambda}_{ML}$	CI/HPD	$\hat{\theta}_S$	$\hat{\theta}_G$	$\hat{\theta}_L$	$\hat{\theta}_{ML}$	CI/HPD
	$EBS_3(.)$	(0.79255)	(0.79230)	(0.79275)			(0.15014)	(0.14628)	(0.15065)		
(30,9)	$\hat{R}_S(.)$	0.19194	0.19215	0.19181	0.19479	(0.24596, 1.13783)	0.48073	0.48580	0.48027	0.49303	(0.02841, 0.16727)
	$B(.)$	(0.69648)	(0.69620)	(0.69669)	(0.69189)		(0.10666)	(0.10301)	(0.10699)	(0.09784)	
	$\hat{R}_{EBS1}(.)$	0.19198	0.19218	0.19185		(0.64198, 0.75135)	0.48068	0.48575	0.48022		(0.03512, 0.17786)
	$EBS_1(.)$	(0.69641)	(0.69612)	(0.69662)			(0.10669)	(0.10304)	(0.10702)		
	$\hat{R}_{EBS2}(.)$	0.19241	0.19262	0.19228			0.48056	0.48564	0.48010		
	$EBS_2(.)$	(0.69609)	(0.69580)	(0.69629)			(0.10678)	(0.10313)	(0.10711)		
	$\hat{R}_{EBS3}(.)$	0.19158	0.19179	0.19145			0.48080	0.48587	0.48034		
	$EBS_3(.)$	(0.69673)	(0.69645)	(0.69694)			(0.10661)	(0.10296)	(0.10694)		
(30,6)	$\hat{R}_S(.)$	0.28659	0.28692	0.28639	0.29050	(0.12579, 1.03098)	0.53727	0.54262	0.53699	0.54439	(0.00956, 0.11479)
	$B(.)$	(0.58245)	(0.58210)	(0.58266)	(0.57839)		(0.06701)	(0.06337)	(0.06721)	(0.06218)	
	$\hat{R}_{EBS1}(.)$	0.28653	0.28687	0.28634		(0.52709, 0.63821)	0.53727	0.54262	0.53699		(0.01005, 0.12021)
	$EBS_1(.)$	(0.58246)	(0.58211)	(0.58267)			(0.06701)	(0.06337)	(0.06720)		
	$\hat{R}_{EBS2}(.)$	0.28649	0.28682	0.28629			0.53729	0.54264	0.53700		
	$EBS_2(.)$	(0.58249)	(0.58214)	(0.58270)			(0.06700)	(0.06336)	(0.06720)		
	$\hat{R}_{EBS3}(.)$	0.28661	0.28695	0.28641			0.53726	0.54261	0.53698		
	$EBS_3(.)$	(0.58243)	(0.58208)	(0.58265)			(0.06702)	(0.06338)	(0.06721)		

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Table 3.3 – *Continued from previous page*

(n,m)		$\hat{\lambda}_S$	$\hat{\lambda}_G$	$\hat{\lambda}_L$	$\hat{\lambda}_{ML}$	CI/HPD	$\hat{\theta}_S$	$\hat{\theta}_G$	$\hat{\theta}_L$	$\hat{\theta}_{ML}$	CI/HPD
(40,24)	$\hat{R}_S(.)$	0.04457	0.04460	0.04456	0.04533	(0.58674, 1.38123)	0.28437	0.28839	0.28336	0.30786	(0.12499, 0.36536)
	$B(.)$	(0.98973)	(0.98957)	(0.98989)	(0.98398)		(0.26679)	(0.26303)	(0.26773)	(0.24518)	
	$\hat{R}_{EBS1}(.)$	0.04459	0.04462	0.04458		(0.94095, 1.03845)	0.28430	0.28832	0.28329		(0.14676, 0.38678)
	$EBS_1(.)$	(0.98969)	(0.98954)	(0.98985)			(0.26685)	(0.26310)	(0.26780)		
	$\hat{R}_{EBS2}(.)$	0.04491	0.04493	0.04489			0.28427	0.28829	0.28326		
	$EBS_2(.)$	(0.98944)	(0.98929)	(0.98960)			(0.26689)	(0.26313)	(0.26784)		
	$\hat{R}_{EBS3}(.)$	0.04434	0.04437	0.04433			0.28433	0.28835	0.28332		
	$EBS_3(.)$	(0.98994)	(0.98979)	(0.99011)			(0.26682)	(0.26306)	(0.26777)		
(40,20)	$\hat{R}_S(.)$	0.11770	0.11780	0.11763	0.12114	(0.49933, 1.27881)	0.41857	0.42210	0.41807	0.43734	(0.09132, 0.28209)
	$B(.)$	(0.79740)	(0.79721)	(0.79755)	(0.79040)		(0.15304)	(0.15032)	(0.15342)	(0.13869)	
	$\hat{R}_{EBS1}(.)$	0.11771	0.11781	0.11764		(0.84803, 0.94363)	0.41857	0.42209	0.41807		(0.10949, 0.30039)
	$EBS_1(.)$	(0.79746)	(0.79727)	(0.79761)			(0.15304)	(0.15032)	(0.15342)		
	$\hat{R}_{EBS2}(.)$	0.11760	0.11770	0.11753			0.41866	0.42218	0.41816		
	$EBS_2(.)$	(0.79786)	(0.79767)	(0.79801)			(0.15297)	(0.15025)	(0.15336)		
	$\hat{R}_{EBS3}(.)$	0.11787	0.11797	0.11779			0.41848	0.42201	0.41798		
	$EBS_3(.)$	(0.79706)	(0.79687)	(0.79721)			(0.15311)	(0.15038)	(0.15349)		

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Table 3.3 – *Continued from previous page*

(n,m)		$\hat{\lambda}_S$	$\hat{\lambda}_G$	$\hat{\lambda}_L$	$\hat{\lambda}_{ML}$	CI/HPD	$\hat{\theta}_S$	$\hat{\theta}_G$	$\hat{\theta}_L$	$\hat{\theta}_{ML}$	CI/HPD
(40,16)	$\hat{R}_S(.)$	0.06843	0.06849	0.06839	0.07039	(0.4058, 1.17501)	0.35422	0.35793	0.35351	0.37614	(0.06251, 0.21487)
	$B(.)$	(0.89571)	(0.89554)	(0.89587)	(0.88907)		(0.20485)	(0.20174)	(0.20545)	(0.18671)	
	$\hat{R}_{EBS1}(.)$	0.06845	0.06851	0.06841		(0.75031, 0.8444)	0.35411	0.35783	0.35340		(0.07666, 0.22974)
	$EBS_1(.)$	(0.89573)	(0.89556)	(0.89588)			(0.20494)	(0.20183)	(0.20554)		
	$\hat{R}_{EBS2}(.)$	0.06832	0.06837	0.06827			0.35412	0.35783	0.35341		
	$EBS_2(.)$	(0.89617)	(0.89600)	(0.89632)			(0.20494)	(0.20182)	(0.20554)		
	$\hat{R}_{EBS3}(.)$	0.06865	0.06871	0.06860			0.35411	0.35782	0.35340		
	$EBS_3(.)$	(0.89528)	(0.89512)	(0.89544)			(0.20495)	(0.20183)	(0.20555)		
(40,12)	$\hat{R}_S(.)$	0.19020	0.19036	0.19009	0.19543	(0.30012, 1.05722)	0.47822	0.48169	0.47789	0.49276	(0.03778, 0.15829)
	$B(.)$	(0.68513)	(0.68492)	(0.68527)	(0.67866)		(0.10847)	(0.10596)	(0.10871)	(0.09804)	
	$\hat{R}_{EBS1}(.)$	0.19027	0.19043	0.19017		(0.63886, 0.73133)	0.47825	0.48172	0.47791		(0.04714, 0.16906)
	$EBS_1(.)$	(0.68506)	(0.68486)	(0.68521)			(0.10844)	(0.10594)	(0.10869)		
	$\hat{R}_{EBS2}(.)$	0.19030	0.19046	0.19019			0.47828	0.48175	0.47795		
	$EBS_2(.)$	(0.68497)	(0.68477)	(0.68512)			(0.10842)	(0.10592)	(0.10867)		
	$\hat{R}_{EBS3}(.)$	0.19028	0.19044	0.19018			0.47822	0.48169	0.47788		
	$EBS_3(.)$	(0.68516)	(0.68495)	(0.68530)			(0.10847)	(0.10596)	(0.10871)		

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Table 3.3 – *Continued from previous page*

(n,m)		$\hat{\lambda}_S$	$\hat{\lambda}_G$	$\hat{\lambda}_L$	$\hat{\lambda}_{ML}$	CI/HPD	$\hat{\theta}_S$	$\hat{\theta}_G$	$\hat{\theta}_L$	$\hat{\theta}_{ML}$	CI/HPD
(40,8)	$\hat{R}_S(.)$	0.28359	0.28385	0.28345	0.28939	(0.186898, 0.97046)	0.53493	0.53859	0.53472	0.54410	(0.01665, 0.10809)
	$B(.)$	(0.58468)	(0.58443)	(0.58484)	(0.57868)		(0.06861)	(0.06611)	(0.06876)	(0.06237)	
	$\hat{R}_{EBS1}(.)$	0.28374	0.28400	0.28360		(0.53677, 0.6326)	0.53492	0.53858	0.53471		(0.02147, 0.11579)
	$EBS_1(.)$	(0.58451)	(0.58425)	(0.58467)			(0.06862)	(0.06612)	(0.06876)		
	$\hat{R}_{EBS2}(.)$	0.28389	0.28414	0.28375			0.53489	0.53855	0.53468		
	$EBS_2(.)$	(0.58456)	(0.58430)	(0.58472)			(0.06864)	(0.06614)	(0.06878)		
	$\hat{R}_{EBS3}(.)$	0.28362	0.28387	0.28347			0.53495	0.53861	0.53474		
	$EBS_3(.)$	(0.58447)	(0.58421)	(0.58462)			(0.06860)	(0.06610)	(0.06875)		

~ *CI - Confidence Interval

~ *HPD - Height Posterior Density

~ *MLE - Maximum Likelihood Estimator

~ *Bayesian estimate (in parenthesis)

TABLE 3.4: Risks of estimators of λ and θ under GELF and LINEX with fixed value $\lambda = 1.1$, $\theta = 0.8$, $\delta = 0.1, p = 0.05, c = 4, \gamma = 3$.

		GELF						LINEX					
(n,m)		$\hat{\lambda}_S$	$\hat{\lambda}_G$	$\hat{\lambda}_L$	$\hat{\theta}_S$	$\hat{\theta}_G$	$\hat{\theta}_L$	$\hat{\lambda}_S$	$\hat{\lambda}_G$	$\hat{\lambda}_L$	$\hat{\theta}_S$	$\hat{\theta}_G$	$\hat{\theta}_L$
	$\hat{R}_{ML}$	0.00982			0.15217			0.00922			0.03609		
(20,12)	$\hat{R}_G(.)$	0.00978	0.00981	0.00979	0.14164	0.18492	0.14325	0.00923	0.00923	0.00922	0.03470	0.03982	0.03492
	$\hat{R}_{EBG1}(.)$	0.00977	0.00981	0.00978	0.14163	0.18491	0.14324	0.00922	0.00923	0.00922	0.03470	0.03982	0.03491
	$\hat{R}_{EBG2}(.)$	0.00973	0.00976	0.00973	0.14163	0.18490	0.14323	0.00919	0.00920	0.00918	0.03470	0.03982	0.03491
	$\hat{R}_{EBG3}(.)$	0.00983	0.00986	0.00984	0.14166	0.18493	0.14326	0.00927	0.00928	0.00927	0.03470	0.03982	0.03492
	$\hat{R}_{ML}$	0.01516			0.21613			0.01282			0.04307		
(20,10)	$\hat{R}_G(.)$	0.01497	0.01503	0.01498	0.20191	0.26791	0.20347	0.01273	0.01276	0.01273	0.04170	0.04686	0.04186
	$\hat{R}_{EBG1}(.)$	0.01498	0.01504	0.01499	0.20183	0.26781	0.20339	0.01274	0.01277	0.01274	0.04169	0.04685	0.04185
	$\hat{R}_{EBG2}(.)$	0.01496	0.01502	0.01497	0.20170	0.26768	0.20326	0.01269	0.01272	0.01270	0.04168	0.04684	0.04183
	$\hat{R}_{EBG3}(.)$	0.01501	0.01507	0.01502	0.20197	0.26796	0.20353	0.01279	0.01282	0.01279	0.04171	0.04686	0.04186
	$\hat{R}_{ML}$	0.02148			0.29597			0.01588			0.04933		
(20,8)	$\hat{R}_G(.)$	0.02133	0.02144	0.02135	0.27911	0.37076	0.28066	0.01579	0.01585	0.01581	0.04818	0.05345	0.04829
	$\hat{R}_{EBG1}(.)$	0.02132	0.02143	0.02134	0.27913	0.37079	0.28068	0.01579	0.01585	0.01580	0.04818	0.05345	0.04829
	$\hat{R}_{EBG2}(.)$	0.02132	0.02143	0.02135	0.27922	0.37089	0.28077	0.01579	0.01585	0.01580	0.04819	0.05346	0.04830
	$\hat{R}_{EBG3}(.)$	0.02132	0.02143	0.02135	0.27906	0.37071	0.28061	0.01579	0.01585	0.01580	0.04818	0.05345	0.04829
	$\hat{R}_{ML}$	0.03312			0.40037			0.02224			0.05501		
(20,6)	$\hat{R}_G(.)$	0.03278	0.03297	0.03282	0.38282	0.53828	0.38438	0.02207	0.02216	0.02208	0.05420	0.05979	0.05427

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Table 3.4 – *Continued from previous page*

		GELF						LINEX					
(n,m)		$\hat{\lambda}_S$	$\hat{\lambda}_G$	$\hat{\lambda}_L$	$\hat{\theta}_S$	$\hat{\theta}_G$	$\hat{\theta}_L$	$\hat{\lambda}_S$	$\hat{\lambda}_G$	$\hat{\lambda}_L$	$\hat{\theta}_S$	$\hat{\theta}_G$	$\hat{\theta}_L$
	$\hat{R}_{EBG1}(.)$	0.03279	0.03298	0.03283	0.38287	0.53832	0.38443	0.02207	0.02217	0.02209	0.05420	0.05980	0.05428
	$\hat{R}_{EBG2}(.)$	0.03277	0.03295	0.03280	0.38281	0.53826	0.38437	0.02205	0.02214	0.02207	0.05420	0.05979	0.05427
	$\hat{R}_{EBG3}(.)$	0.03283	0.03301	0.03287	0.38294	0.53840	0.38451	0.02210	0.02220	0.02212	0.05420	0.05980	0.05428
	$\hat{R}_{ML}$	0.05479			0.55437			0.03211			0.06035		
(20,4)	$\hat{R}_G(.)$	0.05427	0.05464	0.05433	0.54190	0.82765	0.54348	0.03190	0.03206	0.03193	0.06001	0.06521	0.06005
	$\hat{R}_{EBG1}(.)$	0.05430	0.05467	0.05436	0.54193	0.82768	0.54351	0.03191	0.03207	0.03194	0.06001	0.06521	0.06006
	$\hat{R}_{EBG2}(.)$	0.05431	0.05467	0.05437	0.54178	0.82751	0.54335	0.03190	0.03206	0.03193	0.06001	0.06521	0.06005
	$\hat{R}_{EBG3}(.)$	0.05430	0.05467	0.05437	0.54210	0.82787	0.54368	0.03192	0.03208	0.03195	0.06002	0.06521	0.06006
	$\hat{R}_{ML}$	0.00714			0.14851			0.00675			0.03562		
(30,18)	$\hat{R}_G(.)$	0.00703	0.00705	0.00704	0.13266	0.14891	0.13368	0.00668	0.00670	0.00669	0.03345	0.03566	0.03360
	$\hat{R}_{EBG1}(.)$	0.00704	0.00706	0.00704	0.13264	0.14889	0.13366	0.00669	0.00670	0.00669	0.03345	0.03566	0.03359
	$\hat{R}_{EBG2}(.)$	0.00703	0.00706	0.00704	0.13271	0.14896	0.13373	0.00668	0.00669	0.00668	0.03346	0.03567	0.03360
	$\hat{R}_{EBG3}(.)$	0.00705	0.00707	0.00706	0.13258	0.14883	0.13360	0.00671	0.00672	0.00671	0.03344	0.03565	0.03359
	$\hat{R}_{ML}$	0.01308			0.21387			0.01058			0.04286		
(30,15)	$\hat{R}_G(.)$	0.01280	0.01284	0.01281	0.19355	0.21491	0.19453	0.01041	0.01044	0.01041	0.04085	0.04295	0.04096
	$\hat{R}_{EBG1}(.)$	0.01280	0.01284	0.01281	0.19349	0.21485	0.19447	0.01041	0.01043	0.01041	0.04085	0.04295	0.04095
	$\hat{R}_{EBG2}(.)$	0.01281	0.01285	0.01282	0.19343	0.21479	0.19442	0.01043	0.01045	0.01043	0.04084	0.04294	0.04094
	$\hat{R}_{EBG3}(.)$	0.01279	0.01283	0.01280	0.19356	0.21492	0.19454	0.01039	0.01042	0.01040	0.04085	0.04295	0.04096

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Table 3.4 – *Continued from previous page*

		GELF						LINEX					
(n,m)		$\hat{\lambda}_S$	$\hat{\lambda}_G$	$\hat{\lambda}_L$	$\hat{\theta}_S$	$\hat{\theta}_G$	$\hat{\theta}_L$	$\hat{\lambda}_S$	$\hat{\lambda}_G$	$\hat{\lambda}_L$	$\hat{\theta}_S$	$\hat{\theta}_G$	$\hat{\theta}_L$
	$\hat{R}_{ML}$	0.02025			0.29458			0.01514			0.04924		
(30,12)	$\hat{R}_G(.)$	0.01980	0.01988	0.01982	0.26964	0.30356	0.27061	0.01485	0.01489	0.01486	0.04750	0.04975	0.04757
	$\hat{R}_{EBG1}(.)$	0.01982	0.01989	0.01984	0.26964	0.30356	0.27061	0.01486	0.01491	0.01487	0.04750	0.04975	0.04757
	$\hat{R}_{EBG2}(.)$	0.01979	0.01987	0.01981	0.26949	0.30340	0.27046	0.01484	0.01488	0.01485	0.04749	0.04974	0.04756
	$\hat{R}_{EBG3}(.)$	0.01985	0.01993	0.01987	0.26981	0.30373	0.27078	0.01489	0.01493	0.01490	0.04751	0.04976	0.04758
	$\hat{R}_{ML}$	0.03305			0.40040			0.02247			0.05501		
(30,9)	$\hat{R}_G(.)$	0.03246	0.03259	0.03249	0.37267	0.43203	0.37366	0.02215	0.02222	0.02216	0.05371	0.05628	0.05376
	$\hat{R}_{EBG1}(.)$	0.03247	0.03260	0.03250	0.37257	0.43192	0.37356	0.02215	0.02222	0.02217	0.05370	0.05628	0.05375
	$\hat{R}_{EBG2}(.)$	0.03256	0.03269	0.03259	0.37232	0.43166	0.37330	0.02220	0.02227	0.02222	0.05369	0.05627	0.05374
	$\hat{R}_{EBG3}(.)$	0.03240	0.03253	0.03242	0.37284	0.43221	0.37383	0.02211	0.02218	0.02212	0.05372	0.05629	0.05377
	$\hat{R}_{ML}$	0.05524			0.55618			0.03304			0.06040		
(30,6)	$\hat{R}_G(.)$	0.05427	0.05452	0.05431	0.52934	0.67039	0.53037	0.03261	0.03272	0.03263	0.05965	0.06288	0.05968
	$\hat{R}_{EBG1}(.)$	0.05425	0.05450	0.05430	0.52936	0.67040	0.53038	0.03260	0.03272	0.03262	0.05966	0.06289	0.05968
	$\hat{R}_{EBG2}(.)$	0.05424	0.05449	0.05428	0.52941	0.67046	0.53044	0.03260	0.03271	0.03262	0.05966	0.06289	0.05969
	$\hat{R}_{EBG3}(.)$	0.05428	0.05453	0.05433	0.52932	0.67036	0.53035	0.03261	0.03272	0.03263	0.05965	0.06288	0.05968
	$\hat{R}_{ML}$	0.00568			0.14500			0.00551			0.03516		
(40,24)	$\hat{R}_G(.)$	0.00555	0.00556	0.00555	0.12669	0.13661	0.12744	0.00542	0.00543	0.00542	0.03259	0.03401	0.03270

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Table 3.4 – *Continued from previous page*

		GELF						LINEX					
(n,m)		$\hat{\lambda}_S$	$\hat{\lambda}_G$	$\hat{\lambda}_L$	$\hat{\theta}_S$	$\hat{\theta}_G$	$\hat{\theta}_L$	$\hat{\lambda}_S$	$\hat{\lambda}_G$	$\hat{\lambda}_L$	$\hat{\theta}_S$	$\hat{\theta}_G$	$\hat{\theta}_L$
	$\hat{R}_{EBG1}(.)$	0.00555	0.00556	0.00555	0.12663	0.13656	0.12739	0.00543	0.00543	0.00543	0.03258	0.03400	0.03269
	$\hat{R}_{EBG2}(.)$	0.00559	0.00560	0.00559	0.12663	0.13655	0.12738	0.00547	0.00547	0.00547	0.03258	0.03400	0.03269
	$\hat{R}_{EBG3}(.)$	0.00552	0.00554	0.00553	0.12665	0.13658	0.12741	0.00539	0.00540	0.00540	0.03258	0.03401	0.03269
	$\hat{R}_{ML}$	0.00996			0.21070			0.00833			0.04256		
(40,20)	$\hat{R}_G(.)$	0.00963	0.00966	0.00964	0.18727	0.20001	0.18799	0.00810	0.00812	0.00811	0.04019	0.04151	0.04027
	$\hat{R}_{EBG1}(.)$	0.00963	0.00966	0.00964	0.18716	0.19990	0.18788	0.00810	0.00812	0.00811	0.04018	0.04150	0.04026
	$\hat{R}_{EBG2}(.)$	0.00962	0.00965	0.00963	0.18717	0.19991	0.18789	0.00809	0.00811	0.00809	0.04018	0.04150	0.04026
	$\hat{R}_{EBG3}(.)$	0.00966	0.00968	0.00966	0.18717	0.19990	0.18789	0.00813	0.00815	0.00813	0.04018	0.04150	0.04026
	$\hat{R}_{ML}$	0.01836			0.29260			0.01417			0.04911		
(40,16)	$\hat{R}_G(.)$	0.01778	0.01783	0.01779	0.26439	0.28239	0.26510	0.01378	0.01381	0.01379	0.04711	0.04841	0.04716
	$\hat{R}_{EBG1}(.)$	0.01778	0.01784	0.01780	0.26439	0.28239	0.26510	0.01378	0.01381	0.01379	0.04711	0.04841	0.04716
	$\hat{R}_{EBG2}(.)$	0.01776	0.01781	0.01777	0.26453	0.28253	0.26524	0.01377	0.01380	0.01378	0.04712	0.04842	0.04717
	$\hat{R}_{EBG3}(.)$	0.01782	0.01787	0.01783	0.26427	0.28227	0.26499	0.01380	0.01383	0.01380	0.04710	0.04840	0.04715
	$\hat{R}_{ML}$	0.03260			0.39974			0.02256			0.05498		
(40,12)	$\hat{R}_G(.)$	0.03153	0.03163	0.03155	0.36734	0.39717	0.36805	0.02197	0.02202	0.02198	0.05345	0.05486	0.05348
	$\hat{R}_{EBG1}(.)$	0.03155	0.03164	0.03157	0.36740	0.39723	0.36811	0.02198	0.02203	0.02199	0.05345	0.05487	0.05348
	$\hat{R}_{EBG2}(.)$	0.03156	0.03165	0.03158	0.36748	0.39731	0.36819	0.02198	0.02204	0.02199	0.05345	0.05487	0.05349
	$\hat{R}_{EBG3}(.)$	0.03154	0.03164	0.03156	0.36734	0.39717	0.36805	0.02198	0.02203	0.02199	0.05344	0.05486	0.05348

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Table 3.4 – *Continued from previous page*

		GELF						LINEX					
(n,m)		$\hat{\lambda}_S$	$\hat{\lambda}_G$	$\hat{\lambda}_L$	$\hat{\theta}_S$	$\hat{\theta}_G$	$\hat{\theta}_L$	$\hat{\lambda}_S$	$\hat{\lambda}_G$	$\hat{\lambda}_L$	$\hat{\theta}_S$	$\hat{\theta}_G$	$\hat{\theta}_L$
	$\hat{R}_{ML}$	0.05449			0.55504			0.03293			0.06037		
(40,8)	$\hat{R}_G(.)$	0.05308	0.05327	0.05311	0.52098	0.59591	0.52172	0.03229	0.03238	0.03231	0.05941	0.06135	0.05943
	$\hat{R}_{EBG1}(.)$	0.05311	0.05330	0.05315	0.52094	0.59587	0.52168	0.03231	0.03239	0.03233	0.05941	0.06135	0.05943
	$\hat{R}_{EBG2}(.)$	0.05318	0.05337	0.05322	0.52085	0.59577	0.52159	0.03233	0.03241	0.03234	0.05941	0.06135	0.05943
	$\hat{R}_{EBG3}(.)$	0.05305	0.05324	0.05309	0.52104	0.59598	0.52179	0.03230	0.03238	0.03232	0.05941	0.06135	0.05943

TABLE 3.5: Risks of estimators of λ and θ under GELF and LINEX with fixed value $\lambda = 1.1$, $\theta = 0.8$, $\delta = -0.1, p = 0.05, c = 4, \gamma = 3$.

		GELF						LINEX					
(n,m)		$\hat{\lambda}_S$	$\hat{\lambda}_G$	$\hat{\lambda}_L$	$\hat{\theta}_S$	$\hat{\theta}_G$	$\hat{\theta}_L$	$\hat{\lambda}_S$	$\hat{\lambda}_G$	$\hat{\lambda}_L$	$\hat{\theta}_S$	$\hat{\theta}_G$	$\hat{\theta}_L$
(20,12)	$\hat{R}_{ML}$	0.01103			0.22833			0.00978			0.04354		
	$\hat{R}_G(.)$	0.01098	0.01099	0.01097	0.20924	0.22329	0.20641	0.00977	0.00977	0.00977	0.04170	0.04306	0.04142
	$\hat{R}_{EBG1}(.)$	0.01096	0.01098	0.01096	0.20923	0.22327	0.20640	0.00976	0.00977	0.00976	0.04170	0.04306	0.04141
	$\hat{R}_{EBG2}(.)$	0.01091	0.01092	0.01090	0.20922	0.22326	0.20639	0.00972	0.00973	0.00972	0.04170	0.04306	0.04141
	$\hat{R}_{EBG3}(.)$	0.01103	0.01104	0.01102	0.20927	0.22331	0.20644	0.00981	0.00982	0.00981	0.04170	0.04306	0.04142
(20,10)	$\hat{R}_{ML}$	0.01763			0.35392			0.01406			0.05291		
	$\hat{R}_G(.)$	0.01739	0.01742	0.01737	0.32460	0.34499	0.32146	0.01394	0.01395	0.01393	0.05105	0.05236	0.05085
	$\hat{R}_{EBG1}(.)$	0.01740	0.01743	0.01738	0.32442	0.34481	0.32128	0.01395	0.01396	0.01394	0.05104	0.05235	0.05083
	$\hat{R}_{EBG2}(.)$	0.01739	0.01742	0.01737	0.32417	0.34455	0.32103	0.01391	0.01392	0.01390	0.05102	0.05233	0.05082
	$\hat{R}_{EBG3}(.)$	0.01742	0.01745	0.01741	0.32471	0.34511	0.32157	0.01399	0.01400	0.01398	0.05106	0.05237	0.05085
(20,8)	$\hat{R}_{ML}$	0.02600			0.53381			0.01825			0.06154		
	$\hat{R}_G(.)$	0.02581	0.02586	0.02578	0.49367	0.52581	0.49006	0.01815	0.01817	0.01813	0.05994	0.06123	0.05978
	$\hat{R}_{EBG1}(.)$	0.02579	0.02584	0.02576	0.49372	0.52586	0.49011	0.01814	0.01816	0.01813	0.05994	0.06123	0.05979
	$\hat{R}_{EBG2}(.)$	0.02580	0.02584	0.02577	0.49393	0.52608	0.49032	0.01814	0.01817	0.01813	0.05995	0.06124	0.05979
	$\hat{R}_{EBG3}(.)$	0.02580	0.02585	0.02577	0.49356	0.52570	0.48995	0.01814	0.01817	0.01813	0.05993	0.06122	0.05978
(20,6)	$\hat{R}_{ML}$	0.04164			0.80896			0.02621			0.06952		
	$\hat{R}_G(.)$	0.04118	0.04127	0.04113	0.75944	0.81642	0.75512	0.02599	0.02603	0.02596	0.06837	0.06968	0.06827

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Table 3.5 – *Continued from previous page*

		GELF						LINEX					
(n,m)		$\hat{\lambda}_S$	$\hat{\lambda}_G$	$\hat{\lambda}_L$	$\hat{\theta}_S$	$\hat{\theta}_G$	$\hat{\theta}_L$	$\hat{\lambda}_S$	$\hat{\lambda}_G$	$\hat{\lambda}_L$	$\hat{\theta}_S$	$\hat{\theta}_G$	$\hat{\theta}_L$
	$\hat{R}_{EBG1}(.)$	0.04120	0.04128	0.04115	0.75958	0.81656	0.75525	0.02600	0.02604	0.02597	0.06838	0.06969	0.06827
	$\hat{R}_{EBG2}(.)$	0.04117	0.04125	0.04112	0.75942	0.81640	0.75510	0.02597	0.02601	0.02595	0.06837	0.06968	0.06827
	$\hat{R}_{EBG3}(.)$	0.04124	0.04133	0.04119	0.75979	0.81679	0.75547	0.02603	0.02607	0.02600	0.06838	0.06969	0.06828
(20,4)	$\hat{R}_{ML}$	0.07282			1.30357			0.03911			0.07718		
	$\hat{R}_G(.)$	0.07202	0.07219	0.07192	1.25934	1.38146	1.25382	0.03883	0.03890	0.03879	0.07669	0.07798	0.07662
	$\hat{R}_{EBG1}(.)$	0.07206	0.07224	0.07197	1.25945	1.38157	1.25393	0.03885	0.03892	0.03881	0.07669	0.07798	0.07663
	$\hat{R}_{EBG2}(.)$	0.07208	0.07226	0.07199	1.25893	1.38101	1.25341	0.03884	0.03891	0.03881	0.07668	0.07798	0.07662
	$\hat{R}_{EBG3}(.)$	0.07205	0.07223	0.07196	1.26006	1.38222	1.25453	0.03886	0.03893	0.03882	0.07670	0.07799	0.07663
(30,18)	$\hat{R}_{ML}$	0.00793			0.22163			0.00721			0.04291		
	$\hat{R}_G(.)$	0.00779	0.00780	0.00778	0.19329	0.20094	0.19153	0.00713	0.00713	0.00712	0.04006	0.04086	0.03987
	$\hat{R}_{EBG1}(.)$	0.00780	0.00780	0.00779	0.19325	0.20090	0.19148	0.00713	0.00714	0.00713	0.04006	0.04086	0.03987
	$\hat{R}_{EBG2}(.)$	0.00779	0.00780	0.00779	0.19338	0.20103	0.19161	0.00712	0.00713	0.00712	0.04007	0.04087	0.03988
	$\hat{R}_{EBG3}(.)$	0.00781	0.00782	0.00780	0.19315	0.20080	0.19139	0.00715	0.00715	0.00715	0.04005	0.04085	0.03986
(30,15)	$\hat{R}_{ML}$	0.01524			0.34921			0.01192			0.05263		
	$\hat{R}_G(.)$	0.01489	0.01491	0.01488	0.30770	0.31880	0.30574	0.01170	0.01171	0.01169	0.04991	0.05067	0.04977
	$\hat{R}_{EBG1}(.)$	0.01488	0.01490	0.01487	0.30758	0.31868	0.30562	0.01170	0.01171	0.01169	0.04990	0.05066	0.04976
	$\hat{R}_{EBG2}(.)$	0.01490	0.01491	0.01488	0.30747	0.31856	0.30552	0.01172	0.01173	0.01171	0.04989	0.05065	0.04975
	$\hat{R}_{EBG3}(.)$	0.01489	0.01490	0.01487	0.30773	0.31883	0.30577	0.01169	0.01170	0.01168	0.04991	0.05067	0.04977

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Table 3.5 – *Continued from previous page*

		GELF						LINEX					
(n,m)		$\hat{\lambda}_S$	$\hat{\lambda}_G$	$\hat{\lambda}_L$	$\hat{\theta}_S$	$\hat{\theta}_G$	$\hat{\theta}_L$	$\hat{\lambda}_S$	$\hat{\lambda}_G$	$\hat{\lambda}_L$	$\hat{\theta}_S$	$\hat{\theta}_G$	$\hat{\theta}_L$
(30,12)	$\hat{R}_{ML}$	0.02451			0.53047			0.01744			0.06141		
	$\hat{R}_G(.)$	0.02370	0.02374	0.02368	0.47163	0.48883	0.46940	0.01709	0.01711	0.01708	0.05899	0.05973	0.05889
	$\hat{R}_{EBG1}(.)$	0.02372	0.02375	0.02370	0.47163	0.48883	0.46940	0.01710	0.01712	0.01709	0.05899	0.05973	0.05889
	$\hat{R}_{EBG2}(.)$	0.02369	0.02372	0.02367	0.47129	0.48848	0.46906	0.01708	0.01710	0.01707	0.05897	0.05972	0.05888
	$\hat{R}_{EBG3}(.)$	0.02376	0.02380	0.02374	0.47202	0.48923	0.46979	0.01713	0.01715	0.01712	0.05901	0.05975	0.05891
(30,9)	$\hat{R}_{ML}$	0.04120			0.80902			0.02650			0.06952		
	$\hat{R}_G(.)$	0.04040	0.04046	0.04037	0.73141	0.76209	0.72873	0.02610	0.02613	0.02608	0.06768	0.06844	0.06761
	$\hat{R}_{EBG1}(.)$	0.04042	0.04048	0.04039	0.73114	0.76182	0.72847	0.02610	0.02613	0.02609	0.06767	0.06843	0.06761
	$\hat{R}_{EBG2}(.)$	0.04054	0.04060	0.04051	0.73046	0.76111	0.72778	0.02616	0.02619	0.02615	0.06766	0.06842	0.06759
	$\hat{R}_{EBG3}(.)$	0.04032	0.04038	0.04028	0.73189	0.76259	0.72921	0.02605	0.02608	0.02603	0.06769	0.06845	0.06762
(30,6)	$\hat{R}_{ML}$	0.07261			1.31007			0.04012			0.07725		
	$\hat{R}_G(.)$	0.07119	0.07131	0.07113	1.21556	1.28549	1.21203	0.03956	0.03961	0.03953	0.07617	0.07698	0.07613
	$\hat{R}_{EBG1}(.)$	0.07117	0.07129	0.07110	1.21562	1.28555	1.21209	0.03955	0.03960	0.03952	0.07617	0.07698	0.07613
	$\hat{R}_{EBG2}(.)$	0.07113	0.07125	0.07107	1.21581	1.28575	1.21228	0.03955	0.03959	0.03952	0.07618	0.07698	0.07613
	$\hat{R}_{EBG3}(.)$	0.07122	0.07134	0.07115	1.21550	1.28543	1.21197	0.03956	0.03961	0.03954	0.07617	0.07698	0.07613
(40,24)	$\hat{R}_{ML}$	0.00622			0.21525			0.00585			0.04230		
	$\hat{R}_G(.)$	0.00606	0.00606	0.00605	0.18287	0.18810	0.18157	0.00575	0.00575	0.00574	0.03893	0.03951	0.03878

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Table 3.5 – *Continued from previous page*

		GELF						LINEX					
(n,m)		$\hat{\lambda}_S$	$\hat{\lambda}_G$	$\hat{\lambda}_L$	$\hat{\theta}_S$	$\hat{\theta}_G$	$\hat{\theta}_L$	$\hat{\lambda}_S$	$\hat{\lambda}_G$	$\hat{\lambda}_L$	$\hat{\theta}_S$	$\hat{\theta}_G$	$\hat{\theta}_L$
	$\hat{R}_{EBG1}(.)$	0.00606	0.00607	0.00606	0.18277	0.18800	0.18147	0.00575	0.00575	0.00575	0.03892	0.03949	0.03877
	$\hat{R}_{EBG2}(.)$	0.00610	0.00611	0.00610	0.18276	0.18800	0.18147	0.00579	0.00579	0.00579	0.03891	0.03949	0.03877
	$\hat{R}_{EBG3}(.)$	0.00603	0.00604	0.00603	0.18280	0.18803	0.18151	0.00572	0.00572	0.00572	0.03892	0.03950	0.03878
(40,20)	$\hat{R}_{ML}$	0.02168			0.52569			0.01623			0.06123		
	$\hat{R}_G(.)$	0.02096	0.02098	0.02095	0.45956	0.47119	0.45794	0.01576	0.01577	0.01575	0.05845	0.05897	0.05838
	$\hat{R}_{EBG1}(.)$	0.02096	0.02099	0.02095	0.45957	0.47120	0.45795	0.01576	0.01577	0.01575	0.05845	0.05897	0.05838
	$\hat{R}_{EBG2}(.)$	0.02094	0.02096	0.02092	0.45988	0.47151	0.45826	0.01574	0.01575	0.01573	0.05846	0.05898	0.05839
	$\hat{R}_{EBG3}(.)$	0.02101	0.02103	0.02099	0.45930	0.47093	0.45769	0.01578	0.01579	0.01577	0.05844	0.05896	0.05836
(40,16)	$\hat{R}_{ML}$	0.01141			0.34261			0.00932			0.05222		
	$\hat{R}_G(.)$	0.01102	0.01103	0.01101	0.29521	0.30274	0.29378	0.00905	0.00906	0.00905	0.04902	0.04956	0.04891
	$\hat{R}_{EBG1}(.)$	0.01102	0.01103	0.01101	0.29500	0.30253	0.29358	0.00906	0.00906	0.00905	0.04900	0.04954	0.04890
	$\hat{R}_{EBG2}(.)$	0.01101	0.01102	0.01100	0.29502	0.30255	0.29360	0.00904	0.00905	0.00903	0.04900	0.04954	0.04890
	$\hat{R}_{EBG3}(.)$	0.01104	0.01105	0.01103	0.29502	0.30255	0.29360	0.00908	0.00909	0.00908	0.04900	0.04954	0.04890
(40,12)	$\hat{R}_{ML}$	0.04016			0.80713			0.02656			0.06948		
	$\hat{R}_G(.)$	0.03874	0.03878	0.03871	0.71685	0.73706	0.71493	0.02582	0.02584	0.02581	0.06731	0.06783	0.06726
	$\hat{R}_{EBG1}(.)$	0.03876	0.03880	0.03873	0.71702	0.73724	0.71510	0.02583	0.02586	0.02582	0.06731	0.06783	0.06726
	$\hat{R}_{EBG2}(.)$	0.03878	0.03882	0.03875	0.71724	0.73746	0.71531	0.02584	0.02586	0.02582	0.06732	0.06783	0.06727
	$\hat{R}_{EBG3}(.)$	0.03875	0.03879	0.03872	0.71687	0.73708	0.71494	0.02583	0.02586	0.02582	0.06731	0.06783	0.06726

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Table 3.5 – *Continued from previous page*

		GELF						LINEX					
(n,m)		$\hat{\lambda}_S$	$\hat{\lambda}_G$	$\hat{\lambda}_L$	$\hat{\theta}_S$	$\hat{\theta}_G$	$\hat{\theta}_L$	$\hat{\lambda}_S$	$\hat{\lambda}_G$	$\hat{\lambda}_L$	$\hat{\theta}_S$	$\hat{\theta}_G$	$\hat{\theta}_L$
(40,8)	$\hat{R}_{ML}$	0.07113			1.30595			0.03994			0.07721		
	$\hat{R}_G(.)$	0.06910	0.06919	0.06906	1.18680	1.23216	1.18428	0.03911	0.03914	0.03909	0.07582	0.07637	0.07579
	$\hat{R}_{EBG1}(.)$	0.06915	0.06924	0.06910	1.18667	1.23203	1.18414	0.03913	0.03917	0.03911	0.07582	0.07637	0.07579
	$\hat{R}_{EBG2}(.)$	0.06926	0.06935	0.06922	1.18637	1.23172	1.18385	0.03915	0.03919	0.03913	0.07582	0.07637	0.07578
	$\hat{R}_{EBG3}(.)$	0.06905	0.06914	0.06900	1.18704	1.23241	1.18452	0.03911	0.03915	0.03909	0.07582	0.07638	0.07579

TABLE 3.6: PT-II CBRs under different censoring schemes (n,m) with fixed $n=48,\ p=0.05$ for the survival time of multipal myeloma patients.

(n,m)	$(48,28)$		$(48,24)$		$(48,19)$		$(48,14)$		$(48,10)$	
i	x_i	R_i	x_i	R_i	x_i	R_i	x_i	R_i	x_i	R_i
1	1	2	1	1	1	1	1	3	1	2
2	3	1	1	1	1	1	4	4	3	7
3	4	2	4	4	4	4	5	3	6	2
4	5	1	5	2	5	3	8	2	10	2
5	6	2	7	3	8	2	10	0	10	3
6	8	2	10	0	10	2	10	2	12	1
7	10	0	10	0	11	1	12	3	14	3
8	10	1	10	1	12	4	15	2	16	2
9	11	0	12	0	16	2	16	0	18	1
10	12	0	12	0	18	2	17	3	18	15
11	12	2	13	0	18	1	18	1		
12	15	1	14	0	24	0	24	0		
13	16	0	15	2	36	1	36	2		
14	16	0	16	0	40	0	40	9		
15	17	1	17	1	40	1				
16	18	0	18	3	51	0				
17	18	0	24	1	52	0				
18	18	0	40	0	56	0				
19	23	0	40	0	65	4				
20	24	2	40	0						
21	40	0	50	0						
22	40	1	51	1						
23	51	0	56	0						
24	52	0	65	4						
25	56	0								
26	65	0								
27	66	0								
28	76	2								

TABLE 3.7: Bayesian and E-Bayesian estimates of θ, λ under SELF, GELF and LINEX loss function for the survival time of multipal myeloma patients in presence of PT-II CBRs under different censoring schemes (n,m) with fixed $p = 0.05, c = 3, \gamma = 2,$ & $\delta = 1.5$.

(n,m)		$\hat{\theta}_S$	$\hat{\theta}_G$	$\hat{\theta}_L$	$\hat{\theta}_{ML}$	CI/HPD	$\hat{\lambda}_S$	$\hat{\lambda}_G$	$\hat{\lambda}_L$	$\hat{\lambda}_{ML}$	CI/HPD
(48,28)	$B(.)$	4.97790	4.97770	4.97729	3.35291	(0.19974,9.69114)	3.36920	3.36657	3.36389	4.94544	(0.16009,6.86592)
	$EBE_1(.)$	4.99195	4.99175	4.99135			3.37457	3.37194	3.36925		
	$EBE_2(.)$	4.97349	4.97329	4.97289		(4.92289,5.03292)	3.37011	3.36747	3.36479		(3.21202,3.53936)
	$EBE_3(.)$	5.01041	5.00980	5.00980			3.37904	3.37371	3.37371		
(48,24)	$B(.)$	4.64519	4.64508	4.64489	3.54530	(1.47364,7.77677)	3.49896	3.49756	3.49603	4.62521	(1.01918,6.07142)
	$EBE_1(.)$	4.63970	4.63959	4.63940			3.50395	3.50254	3.50100		
	$EBE_2(.)$	4.63885	4.63874	4.63855		(4.60337,4.68164)	3.48095	3.47956	3.47803		(3.37855,3.62834)
	$EBE_3(.)$	4.64054	4.64025	4.64025			3.52694	3.52398	3.52398		
(48,19)	$B(.)$	4.83437	4.83429	4.83415	4.03527	(1.94823,7.677)	3.97063	3.96931	3.96749	4.81261	(1.47684,6.5937)
	$EBE_1(.)$	4.83475	4.83468	4.83454			3.96887	3.96755	3.96573		
	$EBE_2(.)$	4.78439	4.78432	4.78417		(4.79965,4.86622)	3.96895	3.96763	3.96581		(3.84129,4.08844)
	$EBE_3(.)$	4.88512	4.88490	4.88490			3.96879	3.96565	3.96565		
(48,14)	$B(.)$	0.22757	0.22757	0.22757	8.01567	(4.14627,11.88508)	8.09522	8.09380	8.08830	0.22673	(0.05972,0.39374)
	$EBE_1(.)$	0.22742	0.22742	0.22742			8.09056	8.08915	8.08365		
	$EBE_2(.)$	0.22766	0.22766	0.22766		(0.22549,0.22949)	8.15649	8.15506	8.14952		(7.90785,8.28341)
	$EBE_3(.)$	0.22718	0.22718	0.22718			8.02463	8.01778	8.01778		
(48,10)	$B(.)$	0.15889	0.15888	0.15889	6.73828	(2.87992,10.59664)	6.79265	6.79095	6.78571	0.15790	(0.02777,0.28804)

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Table 3.7 – *Continued from previous page*

(n,m)		$\hat{\theta}_S$	$\hat{\theta}_G$	$\hat{\theta}_L$	$\hat{\theta}_{ML}$	CI/HPD	$\hat{\lambda}_S$	$\hat{\lambda}_G$	$\hat{\lambda}_L$	$\hat{\lambda}_{ML}$	CI/HPD
	$EBE_1(.)$	0.15885	0.15885	0.15885			6.79288	6.79117	6.78594		
	$EBE_2(.)$	0.15861	0.15860	0.15861		(0.15741,0.16051)	6.80870	6.80699	6.80175		(6.60566,6.98023)
	$EBE_3(.)$	0.15910	0.15910	0.15910			6.77705	6.77013	6.77013		

TABLE 3.8: Bayesian and E-Bayesian estimates of θ, λ under SELF, GELF and LINEX loss function for the survival time of multipal myeloma patients in presence of PT-II CBRs under different censoring schemes (n,m) with fixed $p = 0.05, c = 4, \gamma = 3$ & $\delta = 1.5$.

(n,m)		$\hat{\theta}_S$	$\hat{\theta}_G$	$\hat{\theta}_L$	$\hat{\theta}_{ML}$	CI/HPD	$\hat{\lambda}_S$	$\hat{\lambda}_G$	$\hat{\lambda}_L$	$\hat{\lambda}_{ML}$	CI/HPD
(48,28)	$B(.)$	4.97873	4.97852	4.97810	3.35291	(0.19974,9.69114)	3.36747	3.36480	3.36208	4.94544	(0.16009,6.86592)
	$EBE_1(.)$	4.98501	4.98479	4.98437			3.36658	3.36390	3.36119		
	$EBE_2(.)$	4.93788	4.93767	4.93725		(4.92452,5.03615)	3.37748	3.37480	3.37207		(3.20363,3.53404)
	$EBE_3(.)$	5.03213	5.03149	5.03149			3.35567	3.35030	3.35030		
(48,24)	$B(.)$	4.64605	4.64595	4.64577	3.54530	(1.47364,7.77677)	3.49849	3.49711	3.49561	4.62521	(1.01918,6.07142)
	$EBE_1(.)$	4.65379	4.65369	4.65351			3.49355	3.49218	3.49068		
	$EBE_2(.)$	4.64542	4.64532	4.64514		(4.60973,4.68632)	3.49260	3.49123	3.48973		(3.37319,3.61746)
	$EBE_3(.)$	4.66216	4.66187	4.66187			3.49450	3.49163	3.49163		
(48,19)	$B(.)$	4.83558	4.83549	4.83532	4.03527	(1.94823,7.677)	3.97225	3.97100	3.96928	4.81261	(1.47684,6.5937)
	$EBE_1(.)$	4.81898	4.81889	4.81872			3.97034	3.96909	3.96737		
	$EBE_2(.)$	4.81865	4.81856	4.81839		(4.79998,4.86935)	3.99570	3.99444	3.99271		(3.84933,4.09073)
	$EBE_3(.)$	4.81931	4.81905	4.81905			3.94497	3.94203	3.94203		
(48,14)	$B(.)$	0.22752	0.22752	0.22752	8.01567	(4.14627,11.88508)	8.09561	8.09418	8.08862	0.22673	(0.05972,0.39374)
	$EBE_1(.)$	0.22766	0.22766	0.22766			8.09541	8.09398	8.08842		
	$EBE_2(.)$	0.22578	0.22577	0.22578		(0.22556,0.22958)	8.07359	8.07216	8.06661		(7.89329,8.26809)
	$EBE_3(.)$	0.22955	0.22955	0.22955			8.11724	8.11022	8.11022		
(48,10)	$B(.)$	0.15880	0.15880	0.15880	6.73828	(2.87992,10.59664)	6.79146	6.78971	6.78434	0.15790	(0.02777,0.28804)

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Table 3.8 – *Continued from previous page*

(n,m)		$\hat{\theta}_S$	$\hat{\theta}_G$	$\hat{\theta}_L$	$\hat{\theta}_{ML}$	CI/HPD	$\hat{\lambda}_S$	$\hat{\lambda}_G$	$\hat{\lambda}_L$	$\hat{\lambda}_{ML}$	CI/HPD
	$EBE_1(.)$	0.15867	0.15866	0.15867			6.79212	6.79037	6.78499		
	$EBE_2(.)$	0.15887	0.15886	0.15887		(0.15732,0.16045)	6.71064	6.70891	6.70360		(6.5913,6.96652)
	$EBE_3(.)$	0.15846	0.15846	0.15846			6.87359	6.86638	6.86638		

TABLE 3.9: Bayesian and E-Bayesian estimates of θ, λ under SELF, GELF and LINEX loss function for the survival time of multipal myeloma patients in presence of PT-II CBRs under different censoring schemes (n,m) with fixed $p = 0.05, c = 3, \gamma = 2,$ & $\delta = -1.5$.

(n,m)		$\hat{\theta}_S$	$\hat{\theta}_G$	$\hat{\theta}_L$	$\hat{\theta}_{ML}$	CI/HPD	$\hat{\lambda}_S$	$\hat{\lambda}_G$	$\hat{\lambda}_L$	$\hat{\lambda}_{ML}$	CI/HPD
(48,28)	$B(.)$	4.97763	4.97767	4.97825	3.35291	(0.19974,9.69114)	3.36879	3.36932	3.37415	4.94544	(0.16009,6.86592)
	$EBE_1(.)$	4.98289	4.98293	4.98351			3.37279	3.37332	3.37815		
	$EBE_2(.)$	4.94617	4.94622	4.94679		(4.92041,5.03161)	3.35562	3.35615	3.36096		(3.20648,3.54029)
	$EBE_3(.)$	5.01961	5.02023	5.02023			3.38995	3.39535	3.39535		
		5.01961	5.02023	5.02023							
(48,24)	$B(.)$	4.64486	4.64488	4.64515	3.54530	(1.47364,7.77677)	3.49790	3.49819	3.50091	4.62521	(1.01918,6.07142)
	$EBE_1(.)$	4.65256	4.65258	4.65285			3.50019	3.50047	3.50320		
	$EBE_2(.)$	4.66275	4.66277	4.66303		(4.60584,4.68041)	3.52939	3.52968	3.53243		(3.37244,3.62082)
	$EBE_3(.)$	4.64237	4.64266	4.64266			3.47098	3.47397	3.47397		
(48,19)	$B(.)$	4.83565	4.83567	4.83588	4.03527	(1.94823,7.677)	3.96978	3.97003	3.97272	4.81261	(1.47684,6.5937)
	$EBE_1(.)$	4.83040	4.83042	4.83063			3.97198	3.97222	3.97492		
	$EBE_2(.)$	4.86059	4.86061	4.86082		(4.80269,4.87026)	3.98614	3.98639	3.98909		(3.8522,4.09438)
	$EBE_3(.)$	4.80021	4.80044	4.80044			3.95781	3.96074	3.96074		
(48,14)	$B(.)$	0.22756	0.22756	0.22756	8.01567	(4.14627,11.88508)	8.09161	8.09189	8.09825	0.22673	(0.05972,0.39374)
	$EBE_1(.)$	0.22775	0.22775	0.22775			8.09047	8.09075	8.09711		
	$EBE_2(.)$	0.22947	0.22947	0.22947		(0.22543,0.22953)	8.06009	8.06036	8.06670		(7.90251,8.26657)
	$EBE_3(.)$	0.22603	0.22603	0.22603			8.12086	8.12751	8.12751		
(48,10)	$B(.)$	0.15884	0.15884	0.15884	6.73828	(2.87992,10.59664)	6.79120	6.79155	6.79820	0.15790	(0.02777,0.28804)

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Table 3.9 – *Continued from previous page*

(n,m)		$\hat{\theta}_S$	$\hat{\theta}_G$	$\hat{\theta}_L$	$\hat{\theta}_{ML}$	CI/HPD	$\hat{\lambda}_S$	$\hat{\lambda}_G$	$\hat{\lambda}_L$	$\hat{\lambda}_{ML}$	CI/HPD
	$EBE_1(.)$	0.15866	0.15866	0.15866			6.79359	6.79394	6.80059		
	$EBE_2(.)$	0.15726	0.15726	0.15726		(0.15747,0.16041)	6.77578	6.77613	6.78276		(6.60755,6.98574)
	$EBE_3(.)$	0.16005	0.16005	0.16005			6.81140	6.81841	6.81841		

TABLE 3.10: Bayesian and E-Bayesian estimates of θ, λ under SELF, GELF and LINEX loss function for the survival time of multipal myeloma patients in presence of PT-II CBRs under different censoring schemes (n,m) with fixed $p = 0.05, c = 4, \gamma = 3$ & $\delta = -1.5$.

(n,m)		$\hat{\theta}_S$	$\hat{\theta}_G$	$\hat{\theta}_L$	$\hat{\theta}_{ML}$	CI/HPD	$\hat{\lambda}_S$	$\hat{\lambda}_G$	$\hat{\lambda}_L$	$\hat{\lambda}_{ML}$	CI/HPD
(48,28)	$B(.)$	4.97886	4.97891	4.97951	3.35291	(0.19974,9.69114)	3.37059	3.37115	3.37622	4.94544	(0.16009,6.86592)
	$EBE_1(.)$	4.97689	4.97694	4.97754			3.36549	3.36605	3.37111		
	$EBE_2(.)$	4.93876	4.93880	4.93940		(4.92215,5.03355)	3.34502	3.34557	3.35061		(3.19464,3.53052)
	$EBE_3(.)$	5.01503	5.01568	5.01568			3.38597	3.39162	3.39162		
(48,24)	$B(.)$	4.64567	4.64569	4.64595	3.54530	(1.47364,7.77677)	3.50011	3.50039	3.50304	4.62521	(1.01918,6.07142)
	$EBE_1(.)$	4.64750	4.64752	4.64778			3.49784	3.49812	3.50076		
	$EBE_2(.)$	4.67659	4.67661	4.67688		(4.61008,4.68416)	3.49251	3.49279	3.49543		(3.38304,3.62501)
	$EBE_3(.)$	4.61841	4.61869	4.61869			3.50317	3.50609	3.50609		
(48,19)	$B(.)$	4.83464	4.83466	4.83488	4.03527	(1.94823,7.677)	3.97320	3.97345	3.97618	4.81261	(1.47684,6.5937)
	$EBE_1(.)$	4.84130	4.84131	4.84153			3.98278	3.98303	3.98577		
	$EBE_2(.)$	4.82344	4.82345	4.82367		(4.80415,4.87216)	3.98320	3.98345	3.98618		(3.84751,4.09427)
	$EBE_3(.)$	4.85916	4.85939	4.85939			3.98237	3.98535	3.98535		
(48,14)	$B(.)$	0.22754	0.22754	0.22754	8.01567	(4.14627,11.88508)	8.09879	8.09908	8.10582	0.22673	(0.05972,0.39374)
	$EBE_1(.)$	0.22724	0.22724	0.22724			8.09821	8.09850	8.10524		
	$EBE_2(.)$	0.22582	0.22583	0.22583		(0.22542,0.2294)	8.09724	8.09753	8.10426		(7.90938,8.28999)
	$EBE_3(.)$	0.22865	0.22865	0.22865			8.09919	8.10621	8.10621		
(48,10)	$B(.)$	0.15888	0.15888	0.15888	6.73828	(2.87992,10.59664)	6.79080	6.79115	6.79804	0.15790	(0.02777,0.28804)

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Table 3.10 – *Continued from previous page*

(n,m)		$\hat{\theta}_S$	$\hat{\theta}_G$	$\hat{\theta}_L$	$\hat{\theta}_{ML}$	CI/HPD	$\hat{\lambda}_S$	$\hat{\lambda}_G$	$\hat{\lambda}_L$	$\hat{\lambda}_{ML}$	CI/HPD
	$EBE_1(.)$	0.15884	0.15885	0.15884			6.79710	6.79746	6.80436		
	$EBE_2(.)$	0.15960	0.15960	0.15960		(0.15735,0.1605)	6.84537	6.84573	6.85267		(6.59068,6.97344)
	$EBE_3(.)$	0.15809	0.15809	0.15809			6.74884	6.75604	6.75604		

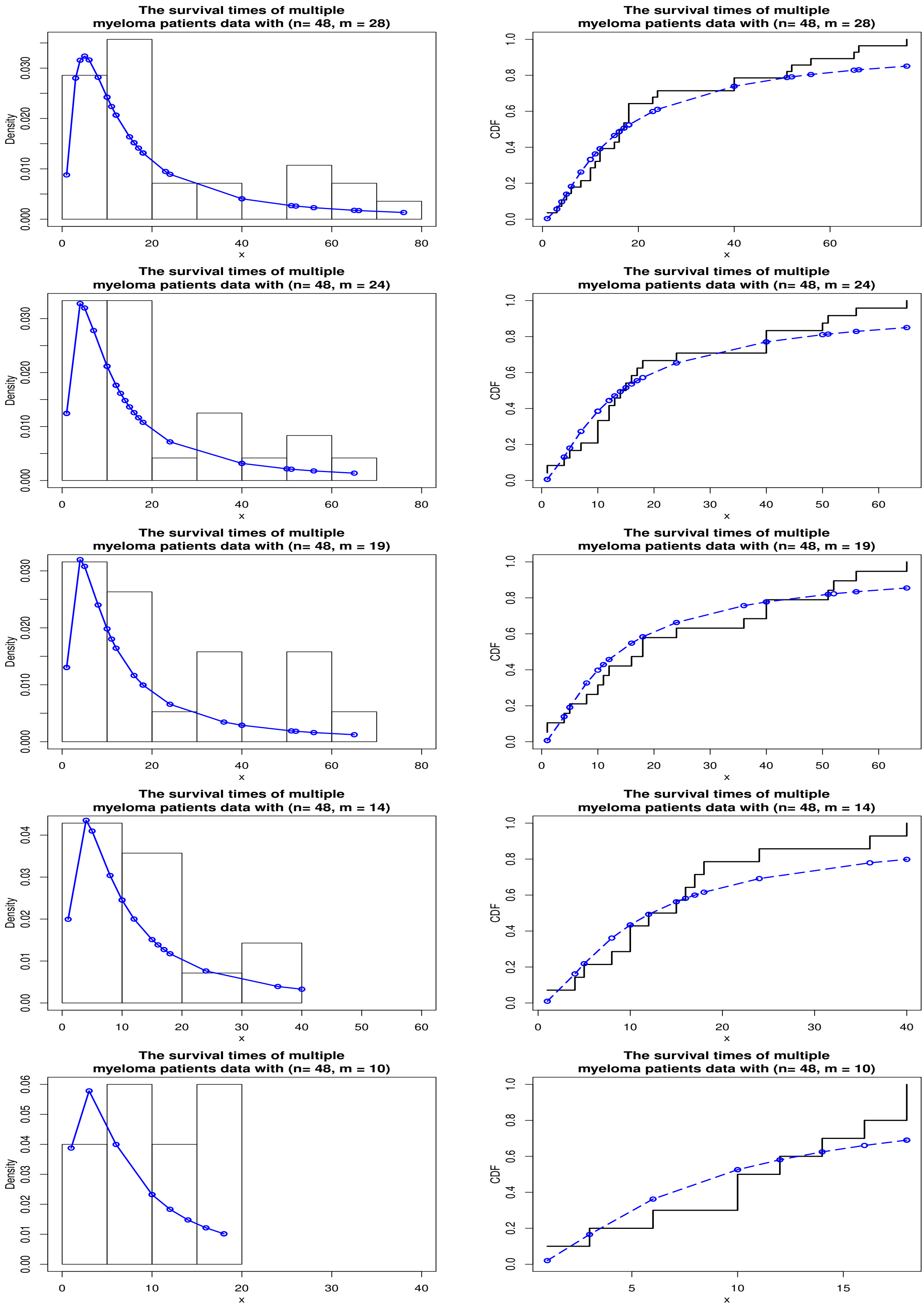

FIGURE 3.4: In the left column is PDF plot and right column is CDF plot for different scheme of the survival time of multipal myeloma patients data.

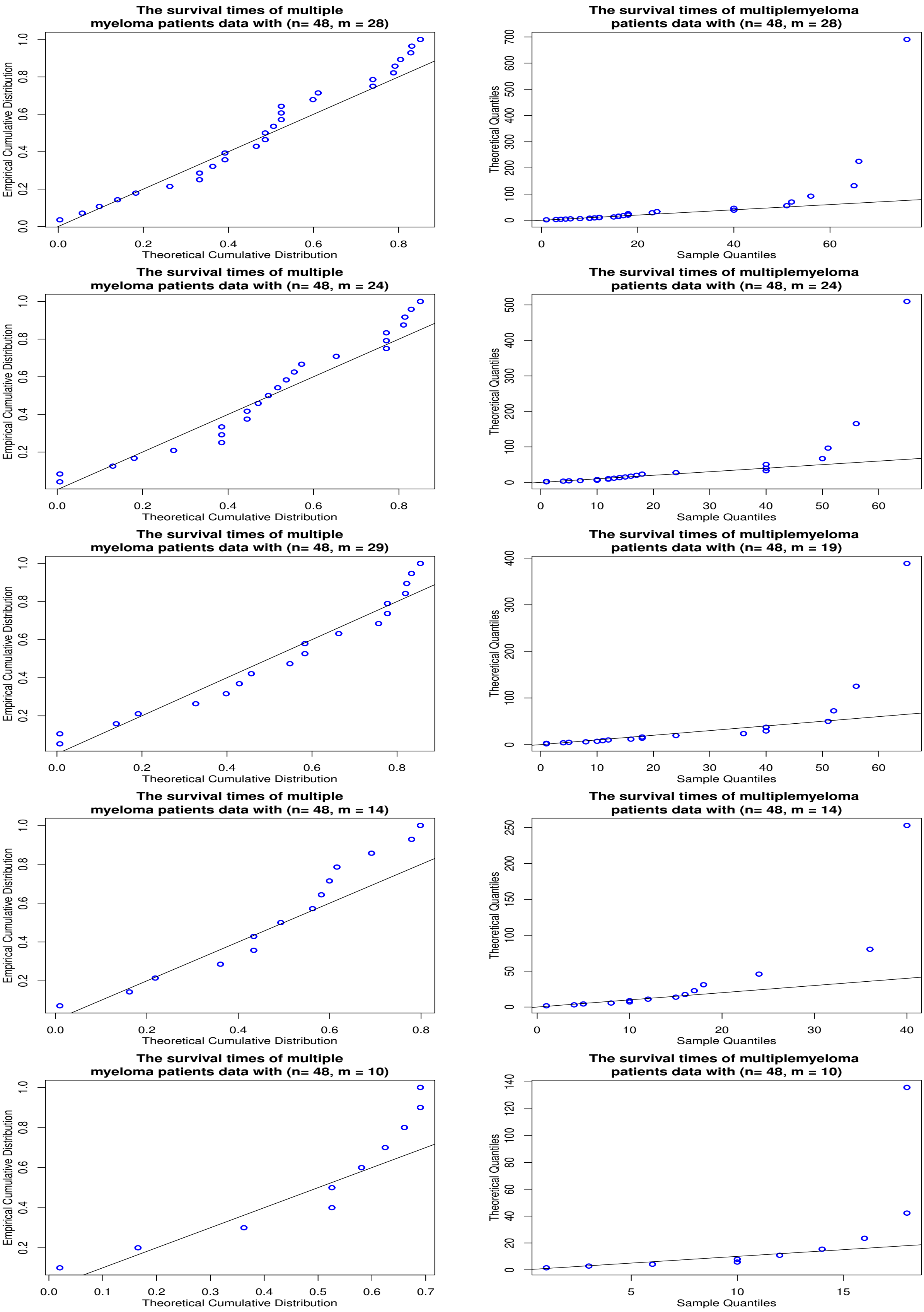

FIGURE 3.5: In the left column is P-P plot and right column is Q-Q plot for different scheme of the survival time of multipal myeloma patients data.

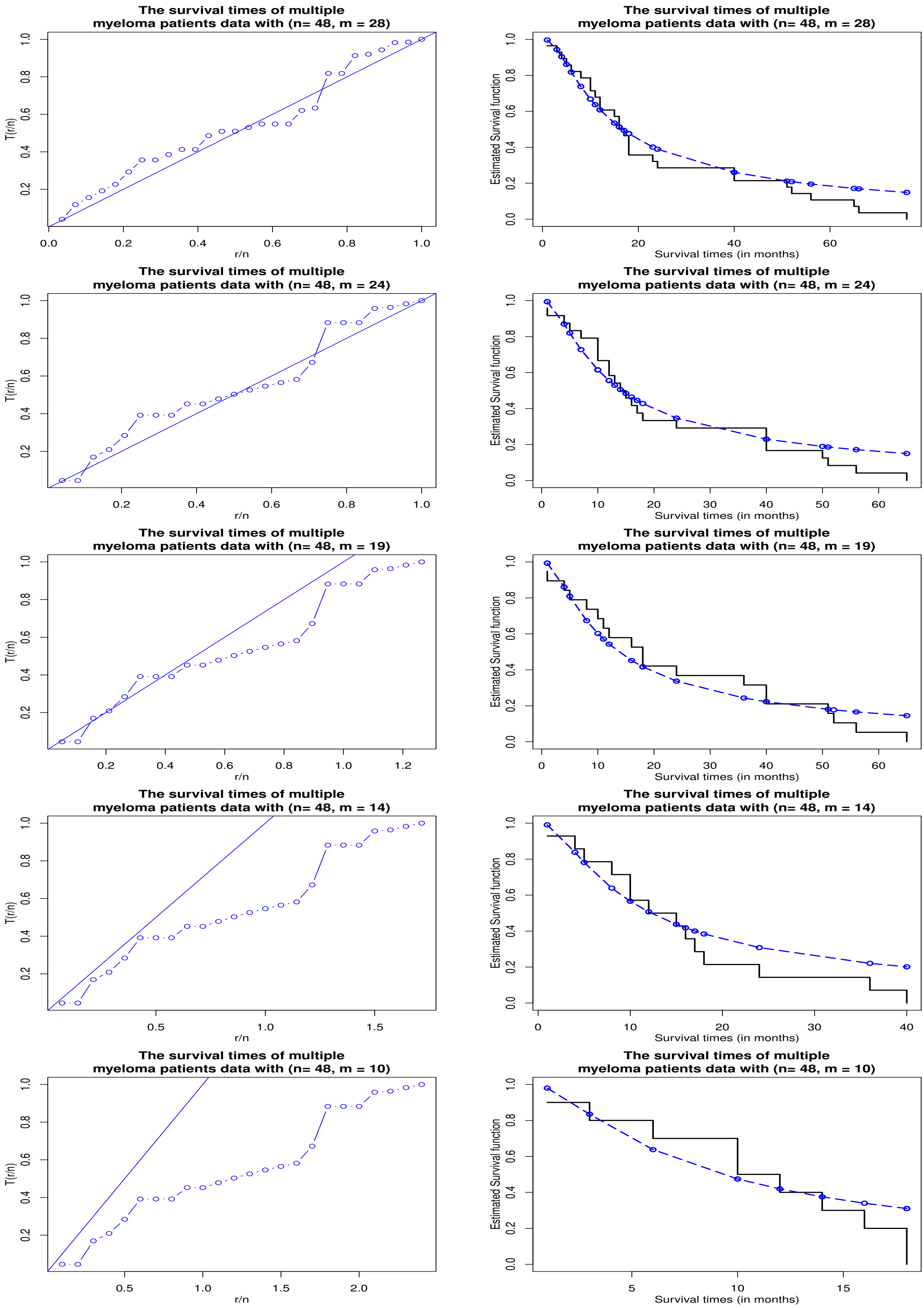

FIGURE 3.6: Left panel is TTT plot and right panel is KM plot for different scheme of the survival time of multipal myeloma patients data.

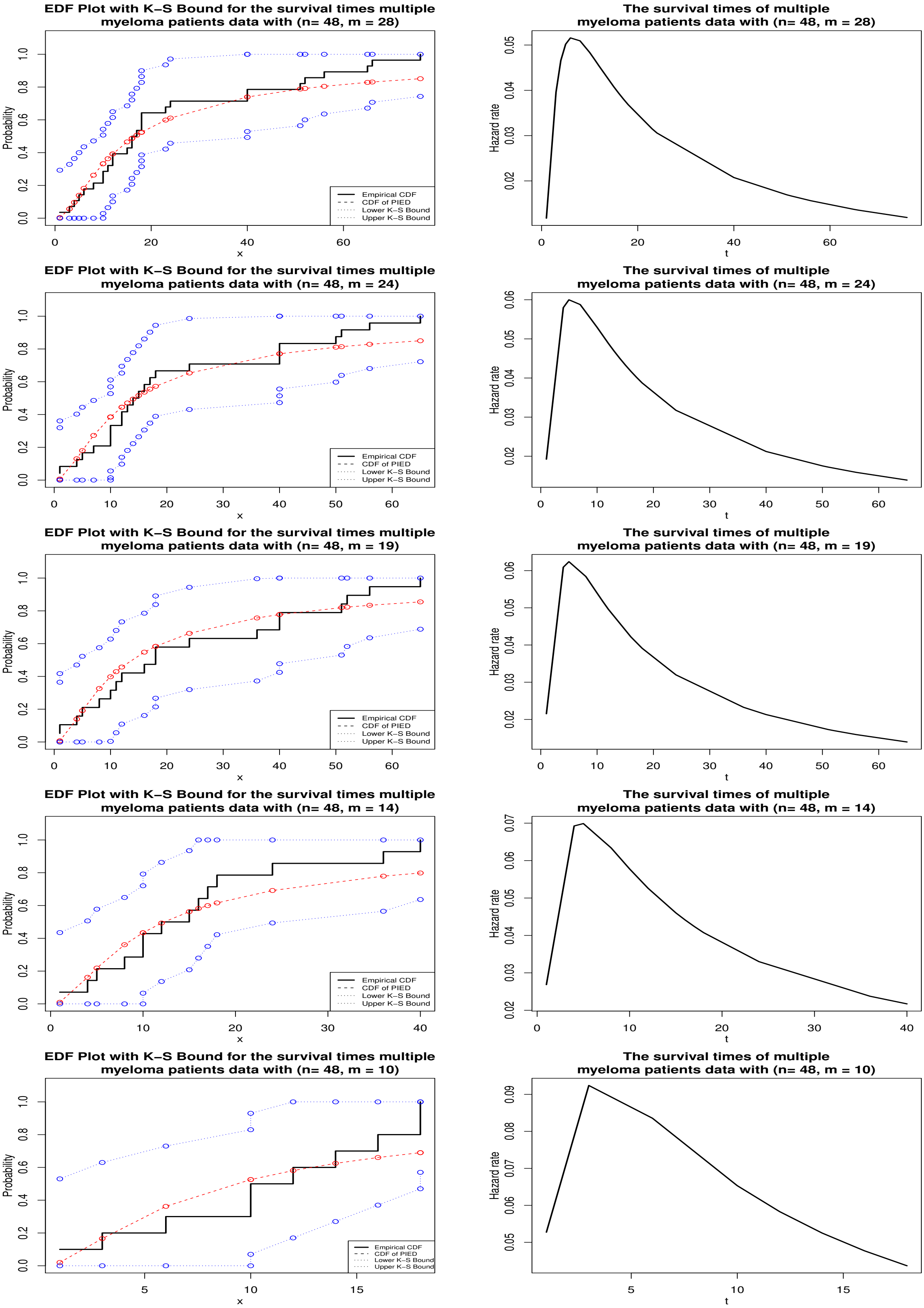

FIGURE 3.7: Left column is K-S plot and and right column is hazard plot for different scheme of the survival time of multipal myeloma patients data.

TABLE 3.11: Quantiles and estimate of λ, θ are obtained for fixed value of $\delta = 0.1$.

n		$\hat{\lambda}$	$\hat{\theta}$	$\xi_{0.05}$	$\xi_{0.25}$	$\xi_{0.50}$	$\xi_{0.75}$	$\xi_{0.95}$	Sample
48	MLE	2.575	3.872	2.213	6.144	13.497	34.210	197.341	113,52,6,40,10,7,
	SELF	2.578	4.172	2.412	6.647	14.538	36.756	211.737	66,10,10,14,16,4,
	GELF	2.476	3.036	1.669	4.659	10.346	26.403	152.919	65,5,11,10,15,5,76,
	LINEX	2.567	3.998	2.287	6.329	13.879	35.140	202.589	56,88,24,51,4,40,
	EBS1	2.574	4.164	2.403	6.624	14.488	36.633	211.031	8,18,5,16,50,40,1,
	EBS2	2.586	4.176	2.423	6.675	14.599	36.909	212.614	36,5,10,91,18,
	EBS3	2.568	4.152	2.390	6.590	14.417	36.457	210.029	1,18,6,1,23,
	EBG1	2.475	3.031	1.666	4.650	10.327	26.355	152.641	15,18,12,
	EBG2	2.484	3.040	1.676	4.678	10.389	26.510	153.536	12,17,3
	EBG3	2.467	3.022	1.656	4.622	10.265	26.199	151.748	
	EBL1	2.566	3.991	2.281	6.316	13.851	35.070	202.193	
	EBL2	2.574	4.003	2.297	6.356	13.937	35.286	203.426	
	EBL3	2.557	3.979	2.266	6.275	13.764	34.855	200.964	
28	MLE	2.677	4.608	2.825	7.677	16.684	42.036	241.688	1,3,4,5,6,8,10,
	SELF	2.778	7.756	5.712	14.125	29.698	73.548	418.925	10,11,12,12,15,
	GELF	2.194	5.033	2.584	6.921	14.953	37.557	215.565	16,16,17,18,18,
	LINEX	2.722	6.965	4.880	12.295	26.009	64.615	368.676	18,23,24,40,
	EBS1	2.782	7.767	5.729	14.165	29.782	73.751	420.075	6,40,51,52,
	EBS2	2.798	7.818	5.811	14.351	30.160	74.676	425.298	56,65,66,7
	EBS3	2.761	7.715	5.638	13.953	29.347	72.688	414.062	
	EBG1	2.194	5.040	2.590	6.934	14.980	37.622	215.935	
	EBG2	2.209	5.073	2.629	7.032	15.184	38.125	218.795	
	EBG3	2.180	5.006	2.551	6.838	14.778	37.123	213.096	
	EBL1	2.723	6.975	4.891	12.318	26.056	64.730	369.323	
	EBL2	2.742	7.021	4.966	12.492	26.414	65.605	374.276	
	EBL3	2.705	6.928	4.817	12.145	25.701	63.860	364.403	
24	MLE	2.749	3.828	2.333	6.483	14.252	36.136	208.496	1,1,4,5,7,10,
	SELF	2.736	4.305	2.658	7.294	15.921	40.210	231.492	10,10,12,12,13,
	GELF	2.563	3.045	1.732	4.836	10.738	27.401	158.688	14,15,16,18,24,
	LINEX	2.717	4.105	2.495	6.888	15.079	38.146	219.809	40,17,18,24,
	EBS1	2.730	4.294	2.643	7.257	15.844	40.019	230.404	40,40,40,50,
	EBS2	2.734	4.301	2.653	7.282	15.897	40.148	231.143	51,56,65

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Table 3.11 – *Continued from previous page*

n		$\hat{\lambda}$	$\hat{\theta}$	$\xi_{0.05}$	$\xi_{0.25}$	$\xi_{0.50}$	$\xi_{0.75}$	$\xi_{0.95}$	Sample
	EBS3	2.730	4.288	2.639	7.245	15.820	39.958	230.063	
	EBG1	2.559	3.038	1.726	4.818	10.698	27.301	158.114	
	EBG2	2.561	3.043	1.730	4.829	10.722	27.361	158.461	
	EBG3	2.557	3.033	1.722	4.807	10.674	27.240	157.768	
	EBL1	2.713	4.095	2.484	6.860	15.020	37.998	218.970	
	EBL2	2.715	4.101	2.491	6.877	15.056	38.088	219.479	
	EBL3	2.711	4.088	2.478	6.842	14.984	37.909	218.461	
19	MLE	2.849	3.543	2.226	6.214	13.714	34.855	201.382	1,1,4,5,8,10,
	SELF	2.847	3.851	2.432	6.755	14.845	37.633	217.109	11,12,16,18,18,
	GELF	2.684	2.734	1.659	4.601	10.242	26.193	151.899	24,36,40,40,51,
	LINEX	2.828	3.692	2.307	6.427	14.155	35.931	207.449	52,56,65
	EBS1	2.834	3.833	2.409	6.692	14.710	37.296	215.186	
	EBS2	2.845	3.841	2.423	6.732	14.795	37.510	216.410	
	EBS3	2.835	3.826	2.404	6.680	14.685	37.234	214.833	
	EBG1	2.677	2.721	1.648	4.571	10.176	26.025	150.933	
	EBG2	2.682	2.727	1.654	4.587	10.212	26.115	151.454	
	EBG3	2.672	2.716	1.643	4.555	10.140	25.934	150.414	
	EBL1	2.820	3.675	2.290	6.380	14.055	35.683	206.037	
	EBL2	2.826	3.682	2.299	6.404	14.108	35.815	206.787	
	EBL3	2.815	3.667	2.281	6.355	14.003	35.553	205.289	
14	MLE	2.276	3.984	2.020	5.592	12.264	31.055	179.050	1,4,5,8,10,
	SELF	2.561	10.365	7.509	17.838	37.001	90.985	516.226	10,12,15,16,17,
	GELF	1.870	6.712	3.196	8.107	17.188	42.750	244.072	18,24,36,40
	LINEX	2.495	8.967	6.142	14.860	31.017	76.521	434.950	
	EBS1	2.563	10.375	7.525	17.874	37.074	91.163	517.232	
	EBS2	2.547	10.323	7.433	17.664	36.647	90.121	511.352	
	EBS3	2.575	10.428	7.605	18.054	37.440	92.053	522.250	
	EBG1	1.870	6.719	3.201	8.117	17.207	42.797	244.336	
	EBG2	1.860	6.684	3.162	8.027	17.023	42.345	241.775	
	EBG3	1.880	6.753	3.239	8.207	17.393	43.252	246.911	
	EBL1	2.495	8.976	6.150	14.877	31.051	76.605	435.422	
	EBL2	2.481	8.930	6.079	14.714	30.719	75.794	430.844	
	EBL3	2.509	9.022	6.222	15.041	31.386	77.420	440.024	

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Table 3.11 – *Continued from previous page*

n		$\hat{\lambda}$	$\hat{\theta}$	$\xi_{0.05}$	$\xi_{0.25}$	$\xi_{0.50}$	$\xi_{0.75}$	$\xi_{0.95}$	Sample
10	MLE	2.112	3.184	1.486	4.154	9.210	23.476	135.872	1,3,6,10,10,
	SELF	3.135	14.758	13.819	31.786	65.178	159.276	900.562	12,14,16,
	GELF	2.125	9.659	5.727	13.720	28.543	70.295	399.177	18,18
	LINEX	3.008	12.085	10.559	24.687	50.923	124.845	707.167	
	EBS1	3.138	14.771	13.845	31.843	65.294	159.558	902.151	
	EBS2	3.124	14.711	13.719	31.561	64.723	158.171	894.339	
	EBS3	3.147	14.831	13.947	32.068	65.748	160.658	908.341	
	EBG1	2.125	9.667	5.733	13.733	28.568	70.355	399.516	
	EBG2	2.117	9.628	5.684	13.622	28.343	69.806	396.418	
	EBG3	2.133	9.707	5.782	13.844	28.795	70.907	402.626	
	EBL1	3.008	12.095	10.569	24.709	50.968	124.953	707.768	
	EBL2	2.997	12.046	10.481	24.511	50.567	123.979	702.280	
	EBL3	3.019	12.144	10.658	24.908	51.371	125.930	713.277	

TABLE 3.12: Quantiles and estimate of λ, θ are obtained for fixed value of $\delta = -0.1$.

n		$\hat{\lambda}$	$\hat{\theta}$	$\xi_{0.05}$	$\xi_{0.25}$	$\xi_{0.50}$	$\xi_{0.75}$	$\xi_{0.95}$	Sample
48	GELF	2.498	3.139	1.735	4.848	10.753	27.420	158.731	13,52,6,40,10,7,
	LINEX	2.699	6.182	4.148	10.684	22.769	56.783	324.659	66,10,10,14,16,4,
	EBG1	2.501	3.146	1.740	4.863	10.786	27.502	159.198	65,5,11,10,15,5,76,
	EBG2	2.511	3.157	1.753	4.898	10.863	27.695	160.311	56,88,24,51,4,40,8,
	EBG3	2.491	3.135	1.728	4.827	10.709	27.309	158.089	18,5,16,50,40,1,36,
	EBL1	2.702	6.195	4.164	10.721	22.845	56.967	325.702	5,10,91,18,1,18,6,
	EBL2	2.713	6.216	4.199	10.805	23.019	57.395	328.125	1,23,15,18,12,
	EBL3	2.692	6.173	4.128	10.638	22.672	56.542	323.288	12,17,3
28	GELF	2.201	5.432	2.860	7.553	16.233	40.662	233.044	1,3,4,5,6,8,10,
	LINEX	3.370	19.748	20.483	46.299	94.313	229.634	1295.705	10,11,12,12,15,
	EBG1	2.202	5.435	2.864	7.561	16.251	40.705	233.287	16,16,17,18,18,
	EBG2	2.219	5.482	2.917	7.689	16.517	41.359	237.000	18,23,24,40,
	EBG3	2.186	5.389	2.811	7.434	15.987	40.056	229.605	40,51,52,56,
	EBL1	3.371	19.762	20.506	46.349	94.414	229.878	1297.075	65,66,76
	EBL2	3.396	19.930	20.851	47.109	95.947	233.589	1317.951	
	EBL3	3.346	19.593	20.164	45.594	92.893	226.196	1276.366	

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Table 3.12 – *Continued from previous page*

n		$\hat{\lambda}$	$\hat{\theta}$	$\xi_{0.05}$	$\xi_{0.25}$	$\xi_{0.50}$	$\xi_{0.75}$	$\xi_{0.95}$	Sample
24	GELF	2.607	3.175	1.830	5.113	11.339	28.905	167.298	1,1,4,5,7,10,10,
	LINEX	2.933	6.654	4.958	12.596	26.720	66.476	379.591	10,12,12,13,
	EBG1	2.609	3.180	1.834	5.124	11.361	28.961	167.617	14,15,16,17,
	EBG2	2.609	3.183	1.836	5.130	11.374	28.993	167.804	18,24,40,40,
	EBG3	2.608	3.177	1.831	5.118	11.348	28.928	167.430	40,50,51,
	EBL1	2.935	6.665	4.971	12.625	26.779	66.619	380.397	56,65
	EBL2	2.936	6.671	4.979	12.642	26.813	66.703	380.866	
	EBL3	2.934	6.659	4.964	12.609	26.745	66.536	379.928	
19	GELF	2.708	2.967	1.789	4.989	11.086	28.305	163.987	1,1,4,5,8,10,
	LINEX	3.061	5.747	4.278	11.177	23.933	59.832	342.548	11,12,16,18,
	EBG1	2.710	2.967	1.791	4.993	11.096	28.330	164.128	18,24,36,40,
	EBG2	2.694	2.949	1.771	4.937	10.972	28.017	162.328	40,51,52,
	EBG3	2.726	2.986	1.811	5.050	11.221	28.645	165.940	56,65
	EBL1	3.063	5.748	4.282	11.187	23.954	59.885	342.853	
	EBL2	3.046	5.712	4.223	11.046	23.663	59.168	338.787	
	EBL3	3.081	5.783	4.342	11.329	24.248	60.606	346.943	
14	GELF	1.916	7.331	3.668	9.157	19.312	47.905	273.105	1,4,5,8,10,10,
	LINEX	3.391	34.354	37.163	82.321	166.354	403.218	2269.300	12,15,16,17,
	EBG1	1.917	7.334	3.670	9.163	19.325	47.936	273.277	18,24,36,
	EBG2	1.918	7.338	3.676	9.175	19.351	47.999	273.639	40
	EBG3	1.915	7.329	3.665	9.150	19.299	47.872	272.916	
	EBL1	3.392	34.366	37.187	82.373	166.460	403.473	2270.734	
	EBL2	3.394	34.388	37.238	82.483	166.681	404.008	2273.741	
	EBL3	3.389	34.344	37.137	82.263	166.238	402.938	2267.730	
10	GELF	2.261	10.173	6.484	15.437	32.045	78.830	447.361	1,3,6,10,10,
	LINEX	4.958	57.511	92.676	203.184	408.862	988.624	5556.204	12,14,16,
	EBG1	2.266	10.212	6.527	15.532	32.238	79.298	449.999	18,18
	EBG2	2.258	10.187	6.487	15.440	32.051	78.840	447.413	
	EBG3	2.274	10.236	6.568	15.625	32.427	79.758	452.593	
	EBL1	4.968	57.733	93.231	204.390	411.280	994.457	5588.938	
	EBL2	4.951	57.595	92.690	203.211	408.912	988.741	5556.842	

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n		$\hat{\lambda}$	$\hat{\theta}$	$\xi_{0.05}$	$\xi_{0.25}$	$\xi_{0.50}$	$\xi_{0.75}$	$\xi_{0.95}$	Sample
	EBL3	4.984	57.871	93.774	205.573	413.654	1000.188	5621.124	