Abstract

The present thesis entitled **"Extension of Some Probability Distributions and Associated Ordered Random Variable"** is being submitted for the award of the degree of Doctor of Philosophy (Statistics). It is based on six chapters and deals mainly with exact expressions and some recurrence relations for the single and product moments of order statistics, best linear unbiased estimators (BLUEs) for the location and scale parameters based on type-II right censored samples and real life applications of some specific continuous distributions. Also, we propose a new model, its statistical and mathematical properties and real life applications. All numerical computations and statistical simulation are performed using R software.

Order statistics and its functions play a vital role in a wide range of theoretical and practical problems such as characterization of probability distributions and goodness-of-fit tests, entropy estimation, analysis of censored samples, reliability analysis, quality control and strength of materials. The moments of order statistics finds wide applicability in many areas such as quality control testing, reliability, etc. For instance, when the reliability of an item or product is high, the duration of the failed items will be high which in turn will make the product too dear, both in terms of time and money. This fact might prevent a practitioner from knowing enough about the product in a relatively short time. Therefore, a practioner needs to predict the failure of future items based on the times of a few early failures. These predictions are often based on moments of order statistics.

Since the turn of this century, order statistics and their moments have gotten a lot of attention. Galton (1902) and Pearson (1902) explored the distribution of the difference of successive order statistics. The moments of order statistics have gained a lot of traction in the statistics field, and they've been numerically tabulated for a variety of distributions. For more information, see David and Nagaraja (2003), Sarhān and Greenberg (1962), Arnold and Balakrishnan (2012), and Arnold et al. (2008).

Chapter 1 is expository in nature and provides a brief review of the concepts and results concerning order statistics, distribution of order statistics, moments and recurrence relations and

some estimation methods. We also discussed few methods to generate distributions. Some continuous distributions, which are used in subsequent chapters, have also been discussed.

Chapter 2 deals with exact explicit expressions for the single and product moments of order statistics from the type II exponentiated log-logistic distribution, and then use these results to compute the means, variances, skewness and kurtosis of uth order statistics. Besides, best linear unbiased estimators (BLUEs) for the location and scale parameters for the type II exponentiated log-logistic distribution with known shape parameters are studied. Finally, the results are illustrated with a real data set.

Chapter 3 deals with exact explicit expressions for the single and product moments of order statistics from the modified power function distribution. By using these relations, we have tabulated the expected values, second moments, variances and covariances of order statistics from samples of sizes up to 10 for various values of the parameters. Also, we use these moments to obtain the best linear unbiased estimates (BLUEs) of the location and scale parameters based on Type-II right-censored samples. In addition, we carry out some numerical illustrations through Monte Carlo simulations to show the usefulness of the findings. Finally, we apply the findings of the chapter to one real data set.

Chapter 4 deals with the inference procedures for the estimation of the parameters by using order statistics. First, we derive some new expressions for the single and product moments of the order statistics from the extended power Lindley distribution. Then, we use these moments to obtain the best linear unbiased estimates (BLUEs) of the location and scale parameters based on Type-II right-censored samples. A real data set are analyzed to illustrate the flexibility and importance of the model.

Chapter 5 deals with recurrence relations (without any restriction for shape parameter) for moments of order statistics from generalized Topp-Leone distribution. Also derived some relations for integral values of shape parameter. These relations will be fruitful for computational purposes. Also, these moments are used to obtain BLUEs of location and scale parameters for

type-II right censored samples. In addition, Monte Carlo simulations is used to find applicability of the findings. Finally, real data set is used to find real life applications.

Chapter 6 deals with a new distribution, based on the WMO-*G* family, using the Lomax distribution as baseline, called Weibull Marshall-Olkin Lomax (WMOL) distribution. This distribution can have different shape of hazard rate function, like unimodal, decreasing, increasing, decreasing-increasing-decreasing and bathtub-shaped. Some properties of proposed model are developed. We also find the maximum likelihood estimates of unknown parameters of the WMOL distribution. For the confirmation of asymptotic behaviour of maximum likelihood estimates we provide simulation study and also used two real data sets to check the applicability of model in real life.