

See discussions, stats, and author profiles for this publication at: <https://www.researchgate.net/publication/348355680>

Erratum Erratum to "Conserved vectors with conformable derivative for certain systems of partial differential equations with physical applications"

Article in *Open Physics* · December 2020

DOI: 10.1515/phys-2020-0219

CITATION

1

READS

54

4 authors:



Pinki Kumari

Central University of Punjab

13 PUBLICATIONS 47 CITATIONS

[SEE PROFILE](#)



Rajesh Kumar Gupta

Central University of Haryana

92 PUBLICATIONS 1,253 CITATIONS

[SEE PROFILE](#)



Sachin Kumar

Central University of Punjab

84 PUBLICATIONS 1,138 CITATIONS

[SEE PROFILE](#)



Maysaa Al Qurashi

Saudi Electronic University

79 PUBLICATIONS 1,926 CITATIONS

[SEE PROFILE](#)

Some of the authors of this publication are also working on these related projects:



fractional differential equations [View project](#)



Solving Fuzzy Equations Using Neural Nets with a New Learning Algorithm and computational approach to approximate fuzzy interpolation polynomials [View project](#)

Erratum

Pinki Kumari, R. K. Gupta, Sachin Kumar*, and Maysaa Mohamed Al Qurashi

Erratum to “Conserved vectors with conformable derivative for certain systems of partial differential equations with physical applications”

10 <https://doi.org/10.1515/phys-2020-0219>
received September 27, 2020; accepted November 27, 2020

Abstract: This erratum corrects the typing mistakes of the article “Conserved vectors with conformable derivative for certain systems of partial differential equations with physical applications,” published in Open Physics 2020;18(1):164–9, <https://doi.org/10.1515/phys-2020-0127>.

Keywords: systems of partial differential equations, Lie symmetry, conformable derivative, conservation laws

1 The dispersive long-wave system

25 In ref. [1], the classical dispersive long-wave system is given by

$$\begin{aligned}\frac{\partial u}{\partial t} &= (u^2 - u_x + 2v)_x, \\ \frac{\partial v}{\partial t} &= (2uv + v_x)_x,\end{aligned}\quad (1)$$

which is used to describe evolution of the horizontal velocity portion of water waves. We present here the correct form of (1) with conformable operator as

$$\begin{aligned}\frac{\partial^\alpha u}{\partial t^\alpha} &= 2u \frac{\partial^\alpha u}{\partial x^\alpha} - \frac{\partial^{2\alpha} u}{\partial x^{2\alpha}} + 2 \frac{\partial^\alpha v}{\partial x^\alpha}, \\ \frac{\partial^\alpha v}{\partial t^\alpha} &= 2u \frac{\partial^\alpha v}{\partial x^\alpha} + 2v \frac{\partial^\alpha u}{\partial x^\alpha} + \frac{\partial^{2\alpha} v}{\partial x^{2\alpha}}.\end{aligned}\quad (2)$$

* **Corresponding author: Sachin Kumar**, Department of Mathematics and Statistics, Central University of Punjab, Punjab, India, e-mail: sachin1jan@yahoo.com

Pinki Kumari: Department of Mathematics and Statistics, Central University of Punjab, Punjab, India, e-mail: yadav.pk1403@gmail.com

R. K. Gupta: Department of Mathematics, Central University of Haryana, Haryana, India, e-mail: rajeshateli@gmail.com

Maysaa Mohamed Al Qurashi: Department of Mathematics, King Saud University, Riyadh, Saudi Arabia, e-mail: maysaa@ksu.edu.sa

We present the symmetries of the correct version of conformable dispersive long-wave system described in (2) as

$$\begin{aligned}\xi^1 &= \frac{c_1 t}{\alpha} + c_2 t^{1-\alpha}, \quad \xi^2 = \frac{c_1}{2\alpha} x + c_4 x^{1-\alpha} + \frac{c_3}{\alpha} x^{1-\alpha} t^\alpha, \\ \eta_1 &= -\frac{c_1}{2} u - \frac{c_3}{2}, \quad \eta_2 = -c_1 v.\end{aligned}\quad (3)$$

Thus, the Lie algebra of (2) is spanned by the following four generators:

$$\begin{aligned}X_1 &= \frac{t}{\alpha} \frac{\partial}{\partial t} + \frac{x}{2\alpha} \frac{\partial}{\partial x} - \frac{1}{2} u \frac{\partial}{\partial u} - v \frac{\partial}{\partial v}, \quad X_2 = t^{1-\alpha} \frac{\partial}{\partial t}, \\ X_3 &= \frac{x^{1-\alpha} t^\alpha}{\alpha} \frac{\partial}{\partial x} - \frac{1}{2} \frac{\partial}{\partial u}, \quad X_4 = x^{1-\alpha} \frac{\partial}{\partial x}.\end{aligned}\quad (4)$$

Below, we will present the correct form of conserved vectors.

For the symmetry $X_1 = \frac{t}{\alpha} \frac{\partial}{\partial t} + \frac{x}{2\alpha} \frac{\partial}{\partial x} - \frac{1}{2} u \frac{\partial}{\partial u} - v \frac{\partial}{\partial v}$, we obtain

$$\begin{aligned}T_1^t &= c_1 x^{\alpha-1} \left(-\frac{1}{2} u - \frac{x}{2\alpha} u_x - \frac{t}{\alpha} u_t \right) \\ &\quad + c_2 x^{\alpha-1} \left(-v - \frac{x}{2\alpha} v_x - \frac{t}{\alpha} v_t \right), \\ T_1^x &= 2 \left(-\frac{1}{2} u - \frac{x}{2\alpha} u_x - \frac{t}{\alpha} u_t \right) (c_1 u - c_2 v) t^{1-\alpha} \\ &\quad - c_1 \left(\frac{1}{2} u_x + \frac{x}{2\alpha} u_{xx} + \frac{1}{2\alpha} u_x + \frac{1}{\alpha} u_{tx} \right) x^{1-\alpha} t^{\alpha-1} \\ &\quad - \left(-v - \frac{x}{2\alpha} v_x - \frac{t}{\alpha} v_t \right) (2c_1 + 2c_2 u + c_2 x^{-\alpha} \\ &\quad + c_2 (1 - \alpha) x^{-\alpha}) t^{\alpha-1} \\ &\quad + c_2 \left(v_x + \frac{x}{2\alpha} v_{xx} + \frac{1}{2\alpha} v_x + \frac{1}{\alpha} v_{tx} \right) x^{1-\alpha} t^{\alpha-1}.\end{aligned}\quad (5)$$

For the symmetry $X_2 = t^{1-\alpha} \frac{\partial}{\partial t}$, we obtain

$$\begin{aligned}T_1^t &= -c_1 x^{\alpha-1} t^{1-\alpha} u_t - c_2 x^{\alpha-1} t^{1-\alpha} v_t, \\ T_1^x &= -t^{1-\alpha} u_t (c_1 u - c_2 v) t^{1-\alpha} \\ &\quad - c_1 t^{1-\alpha} u_{tx} x^{1-\alpha} t^{\alpha-1} + t^{1-\alpha} v_t (2c_1 + 2c_2 u \\ &\quad + c_2 x^{-\alpha} + c_2 (1 - \alpha) x^{-\alpha}) t^{\alpha-1} + c_2 v_{tx} x^{1-\alpha}.\end{aligned}\quad (6)$$

For the symmetry $X_3 = \frac{x^{1-\alpha} t^\alpha}{\alpha} \frac{\partial}{\partial x} - \frac{1}{2} \frac{\partial}{\partial u}$, we obtain

$$\begin{aligned}
 T_1^t &= c_1 x^{\alpha-1} \left(-\frac{1}{2} - \frac{x^{1-\alpha} t^\alpha}{\alpha} u_x \right) - c_2 \frac{t^\alpha}{\alpha} v_x, \\
 T_1^x &= \left(-\frac{1}{2} - \frac{x^{1-\alpha} t^\alpha}{\alpha} u_x \right) (c_1 u - c_2 v) t^{1-\alpha} \\
 &\quad - c_1 \left(\frac{(1-\alpha)}{\alpha} u_x + \frac{1}{\alpha} x u_{xx} \right) x^{1-2\alpha} t^{2\alpha-1} \\
 &\quad + \frac{x^{1-\alpha} t^\alpha}{\alpha} v_x (2c_1 + 2c_2 u + c_2 x^{-\alpha} \\
 &\quad + c_2 (1-\alpha) x^{-\alpha}) t^{\alpha-1} \\
 &\quad + c_2 \left(\frac{(1-\alpha)}{\alpha} v_x + \frac{1}{\alpha} x v_{xx} \right) x^{1-2\alpha} t^{2\alpha-1}.
 \end{aligned} \tag{7}$$

For the symmetry $X_4 = x^{1-\alpha} \frac{\partial}{\partial x}$, we obtain

$$\begin{aligned}
 T_1^t &= -c_1 u_x - c_2 v_x, \\
 T_1^x &= -x^{1-\alpha} u_x (c_1 u - c_2 v) t^{1-\alpha} - c_1 ((1-\alpha) u_x \\
 &\quad + x u_{xx}) x^{1-2\alpha} t^{\alpha-1} + x^{1-\alpha} v_x (2c_1 + 2c_2 u \\
 &\quad + c_2 x^{-\alpha} + c_2 (1-\alpha) x^{-\alpha}) t^{\alpha-1} \\
 &\quad + c_2 ((1-\alpha) v_x + x v_{xx}) x^{1-2\alpha} t^{\alpha-1}.
 \end{aligned} \tag{8}$$

2 The Whitham–Broer–Kaup system

The conformable Whitham–Broer–Kaup Wilson system used in ref. [1] is given by

$$\begin{aligned}
 \frac{\partial^\alpha u}{\partial t^\alpha} + u \frac{\partial^\alpha u}{\partial x^\alpha} + \mu \frac{\partial^{2\alpha} u}{\partial x^{2\alpha}} + \frac{\partial^\alpha v}{\partial x^\alpha} &= 0, \\
 \frac{\partial^\alpha v}{\partial t^\alpha} + u \frac{\partial^\alpha v}{\partial x^\alpha} + v \frac{\partial^\alpha u}{\partial x^\alpha} - \mu \frac{\partial^{2\alpha} v}{\partial x^{2\alpha}} + \beta \frac{\partial^{3\alpha} v}{\partial x^{3\alpha}} &= 0.
 \end{aligned} \tag{9}$$

We present the following symmetries for system (9)

$$\begin{aligned}
 \xi^1 &= \frac{c_1 t}{\alpha} + c_2 t^{1-\alpha}, \quad \xi^2 = \frac{c_1}{2\alpha} x + c_4 x^{1-\alpha} + \frac{c_3}{\alpha} x^{1-\alpha} t^\alpha, \\
 \eta_1 &= -\frac{c_1}{2} u + c_3, \quad \eta_2 = -c_1 v.
 \end{aligned} \tag{10}$$

Thus, the Lie algebra of (9) is spanned by the following four generators

$$\begin{aligned}
 X_1 &= \frac{t}{\alpha} \frac{\partial}{\partial t} + \frac{x}{2\alpha} \frac{\partial}{\partial x} - \frac{1}{2} u \frac{\partial}{\partial u} - v \frac{\partial}{\partial v}, \quad X_2 = t^{1-\alpha} \frac{\partial}{\partial t}, \\
 X_3 &= \frac{x^{1-\alpha} t^\alpha}{\alpha} \frac{\partial}{\partial x} + \frac{\partial}{\partial u}, \quad X_4 = x^{1-\alpha} \frac{\partial}{\partial x}.
 \end{aligned} \tag{11}$$

In the next step, we present the correct version of the conserved vectors for system (9).

For the symmetry $X_1 = \frac{t}{\alpha} \frac{\partial}{\partial t} + \frac{x}{2\alpha} \frac{\partial}{\partial x} - \frac{1}{2} u \frac{\partial}{\partial u} - v \frac{\partial}{\partial v}$, we obtain

$$\begin{aligned}
 T_1^t &= c_1 x^{\alpha-1} \left(-\frac{1}{2} u - \frac{x}{2\alpha} u_x - \frac{t}{\alpha} u_t \right) \\
 &\quad + c_2 x^{\alpha-1} \left(-v - \frac{x}{2\alpha} v_x - \frac{t}{\alpha} v_t \right), \\
 T_1^x &= \left(-\frac{1}{2} u - \frac{x}{2\alpha} u_x - \frac{t}{\alpha} u_t \right) (c_1 u + c_1 \mu c_2 x^{-\alpha} + c_2 v \\
 &\quad + c_1 \beta (1-\alpha) (1-2\alpha) x^{-2\alpha} - c_1 \mu (1-\alpha) x^{-\alpha} \\
 &\quad - 3c_2 \beta (1-\alpha)^2 x^{-\alpha} + c_2 \beta (2-2\alpha) (1-\alpha) x^{-\alpha}) t^{\alpha-1} \\
 &\quad - \left(\frac{1}{2} u_x + \frac{x}{2\alpha} u_{xx} + \frac{1}{2\alpha} u_x + \frac{1}{\alpha} u_{tx} \right) \\
 &\quad \times (c_1 \mu x^{1-\alpha} + 3\beta (1-\alpha) c_2 x^{1-2\alpha} - c_2 \beta x^{2-2\alpha}) t^{\alpha-1} \\
 &\quad - \left(\frac{1}{2} u_{xx} + \frac{1}{2\alpha} u_{xx} + \frac{x}{2\alpha} u_{xxx} + \frac{1}{2\alpha} u_{xx} + \frac{1}{\alpha} u_{txx} \right) \\
 &\quad \times c_2 x^{2-2\alpha} t^{\alpha-1} + \left(-v - \frac{x}{2\alpha} v_x - \frac{t}{\alpha} v_t \right) \\
 &\quad \times (c_1 + c_2 u - c_2 \mu (1-\alpha) x^{1-\alpha} + c_2 \mu (1-\alpha) x^{-\alpha}) t^{\alpha-1} \\
 &\quad - \left(v_x + \frac{x}{2\alpha} v_{xx} + \frac{1}{2\alpha} v_x + \frac{1}{\alpha} v_{tx} \right) c_2 \mu x^{1-\alpha} t^{\alpha-1}.
 \end{aligned} \tag{12}$$

For the symmetry $X_2 = t^{1-\alpha} \frac{\partial}{\partial t}$, we obtain

$$\begin{aligned}
 T_1^t &= -c_1 x^{\alpha-1} t^{1-\alpha} u_t - c_2 x^{\alpha-1} t^{1-\alpha} v_t, \\
 T_1^x &= -u_t (c_1 u + c_1 \mu c_2 x^{-\alpha} + c_2 v + c_1 \beta (1-\alpha) \\
 &\quad \times (1-2\alpha) x^{-2\alpha} - c_1 \mu (1-\alpha) x^{-\alpha} \\
 &\quad - 3c_2 \beta (1-\alpha)^2 x^{-\alpha} \\
 &\quad + c_2 \beta (2-2\alpha) (1-\alpha)) x^{-\alpha} - u_{tx} (c_1 \mu x^{1-\alpha} \\
 &\quad + 3\beta (1-\alpha) c_2 x^{1-2\alpha} - c_2 \beta x^{2-2\alpha}) - u_{txx} c_2 x^{2-2\alpha} \\
 &\quad - v_t (c_1 + c_2 u - c_2 \mu (1-\alpha) x^{1-\alpha} \\
 &\quad + c_2 \mu (1-\alpha) x^{-\alpha}) - c_2 \mu x^{1-\alpha} v_{tx}.
 \end{aligned} \tag{13}$$

For the symmetry $X_3 = \frac{x^{1-\alpha} t^\alpha}{\alpha} \frac{\partial}{\partial x} + \frac{\partial}{\partial u}$, we obtain

$$\begin{aligned}
 T_1^t &= c_1 x^{\alpha-1} \left(-1 - \frac{x^{1-\alpha} t^\alpha}{\alpha} u_x \right) - c_2 \frac{t^\alpha}{\alpha} v_x, \\
 T_1^x &= \left(-1 - \frac{x^{1-\alpha} t^\alpha}{\alpha} u_x \right) (c_1 u + c_1 \mu c_2 x^{-\alpha} + c_2 v \\
 &\quad + c_1 \beta (1-\alpha) (1-2\alpha) x^{-2\alpha} - c_1 \mu (1-\alpha) x^{-\alpha} \\
 &\quad - 3c_2 \beta (1-\alpha)^2 x^{-\alpha} + c_2 \beta (2-2\alpha) (1-\alpha)) x^{-\alpha} t^{\alpha-1} \\
 &\quad - \left(\frac{(1-\alpha)}{\alpha} u_x + \frac{1}{\alpha} x u_{xx} \right) (c_1 \mu x^{1-\alpha} + 3\beta (1-\alpha) c_2 x^{1-2\alpha} \\
 &\quad - c_2 \beta x^{2-2\alpha}) t^{2\alpha-2} - \left(\frac{(1-\alpha)}{\alpha} u_{xx} + \frac{1}{\alpha} x u_{xxx} + \frac{1}{\alpha} u_{xx} \right) \\
 &\quad \times c_2 x^{2-2\alpha} t^{\alpha-1} + \frac{x^{1-\alpha} t^\alpha}{\alpha} v_x (2c_1 + 2c_2 u \\
 &\quad + c_2 x^{-\alpha} + c_2 (1-\alpha) x^{-\alpha}) t^{\alpha-1} \\
 &\quad + c_2 \mu \left(\frac{(1-\alpha)}{\alpha} v_x + \frac{1}{\alpha} x v_{xx} \right) x^{1-2\alpha} t^{2\alpha-1}.
 \end{aligned} \tag{14}$$

For the symmetry $X_4 = x^{1-\alpha} \frac{\partial}{\partial x}$, we obtain

5

10

15

20

25

30

35

40

45

50

5

10

15

20

25

30

35

40

45

50

$$T_1^t = -c_1 u_x - c_2 v_x,$$

$$\begin{aligned} T_1^x = & -x^{1-\alpha} u_x (c_1 u + c_1 \mu c_2 x^{-\alpha} + c_2 v + c_1 \beta (1 - \alpha) \\ & \times (1 - 2\alpha) x^{-2\alpha} - c_1 \mu (1 - \alpha) x^{-\alpha} - 3c_2 \beta (1 - \alpha)^2 \\ & \times x^{-\alpha} + c_2 \beta (2 - 2\alpha) (1 - \alpha) x^{-\alpha}) t^{\alpha-1} \\ & - ((1 - \alpha) x^{-\alpha} u_x + x^{1-\alpha} u_{xx}) \\ & \times (c_1 \mu x^{1-\alpha} + 3\beta (1 - \alpha) c_2 x^{1-2\alpha} - c_2 \beta x^{2-2\alpha}) t^{\alpha-1} \quad (15) \\ & - (2(1 - \alpha) x^{-\alpha} u_{xx} - \alpha (1 - \alpha) x^{-\alpha-1} u_x \\ & + x^{1-\alpha} u_{xxx}) c_2 x^{2-2\alpha} t^{\alpha-1} - x^{1-\alpha} v_x (c_1 + c_2 u \\ & - c_2 \mu (1 - \alpha) x^{1-\alpha} + c_2 \mu (1 - \alpha) x^{-\alpha}) t^{\alpha-1} \\ & - ((1 - \alpha) x^{-\alpha} v_x + x^{1-\alpha} v_{xx}) c_2 \mu x^{1-\alpha} t^{\alpha-1}. \end{aligned}$$

References

- [1] Al Qurashi MM. Conserved vectors with conformable derivative for certain systems of partial differential equations with physical applications. *Open Phys.* 2020;18(1):164–9.

5

10